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The various contributions in Venus rotation rate and LOD

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ABSTRACT

Context. Thanks to the Venus Express Mission, new data on the properties of Venus could be obtained, in particular concerning its rotation.

Aims. In view of these upcoming results, the purpose of this paper is to determine and compare the major physical processes influencing the rotation of Venus and, more particularly, the angular rotation rate.

Methods. Applying models already used for Earth, the effect of the triaxiality of a rigid Venus on its period of rotation are computed. Then the variations of Venus rotation caused by the elasticity, the atmosphere, and the core of the planet are evaluated.

Results. Although the largest irregularities in the rotation rate of the Earth on short time scales are caused by its atmosphere and elastic deformations, we show that the irregularities for Venus are dominated by the tidal torque exerted by the Sun on its solid body. Indeed, as Venus has a slow rotation, these effects have a large amplitude of two minutes of time (mn). These variations in the rotation rate are greater than the one induced by atmospheric wind variations that can reach 25–50 s of time (s), depending on the simulation used. The variations due to the core effects that vary with its size between 3 and 20 s are smaller. Compared to these effects, the influence of the elastic deformation caused by the zonal tidal potential is negligible.

Conclusions. As the variations in the rotation of Venus reported here are close to 3 mn peak to peak, they should influence past, present, and future observations, thereby providing further constraints on the planet's internal structure and atmosphere.

Key words. celestial mechanics – planets and satellites: individual: Venus

1. Introduction

The study of the irregularities in the rotation of a planet provides astronomers and geophysicists with physical contraints on the models describing this planet and leads to better understanding of its global properties. Although Venus is the planet that shares the most similarities with the Earth in terms of size and density, some of its characteristics are poorly understood, such as its atmospheric winds and its superrotation. With its thick atmosphere, determinating the period of rotation of Venus has been and still is a very challenging task. Today, thanks to Venus Express (the first spacecraft orbiting Venus since the Magellan Mission in 1994) and Earth-based radar measurements, the period of rotation of Venus can be revised and its variations evaluated, so here for the first time we present a complete theoretical study of the variations in the rotation of Venus on a short time scale, as well as its implications for the observations.

There have already been many studies on the rotation of Venus. Several authors have studied this rotation on a long time scale to understand why Venus' spin is retrograde and why its spin axis has low obliquity (Goldstein 1964; Carpenter 1964; Goldreich & Peale 1970; Lago & Cazenave 1979; Dobrovoskis 1980; Yoder 1995; Correia & Laskar 2001, 2003). Others studied the possible resonance between the Earth and Venus (Gold & Soter 1979; Bills 2005; Bazsó et al. 2010). But few studies have been made comparing the major physical processes influencing the rotation of Venus at short time scale. Karatekin et al. (2010) show that the variations in the rotation of Venus due to the atmosphere should be approximatively ten seconds for the 117 day time span characterizing the planet's solar day, but they neglected the main effect of the impact of gravitational torques on the solid body of the planet. Here, applying different models already used for the Earth, the irregularities in the rotation of Venus are computed. After clarifying the link between the length of day (LOD) and the rotation rate of the planet, the effects of the triaxiality of a rigid Venus on its period of rotation are evaluated. This model is based on Kinoshita's theory (1977) and uses the Andoyer variables (Andoyer 1923). Because Venus has a very slow rotation (–243.020 d), these effects have a large amplitude (2 mn peak to peak) and should be observed as shown in Sect. 3.

As we know, the variation in the period of rotation of the Earth of about 0.6 ms (ms) for the seasonal component, are mainly caused by the tidal zonal potential, the atmosphere, and the oceans (Munk & MacDonald 1960; Lambeck 1980; Barnes et al. 1983). Hereafter quantifying the effect of the zonal component of the solar tides on Venus (Sect. 4), the effects of its atmosphere (Sect. 5), and the impact of its internal structure (Sect. 6) are presented.

To conclude, a comparison of the properties and amplitudes of these various processes is analyzed (Sect. 7). Their observability and their implications concerning the properties of Venus are also discussed. We find that, in contrast to what is often assumed, the atmosphere should not be the most important cause of variations of in rotation rate of Venus. Indeed, the tidal torque exerted by the Sun on the solid body of the planet should be greater by more than a factor of two.

2. Rotation rate and LOD definition

A day is defined as the time between two consecutive crossings of a reference meridian by a reference point or body. While these periods vary with respect to time, we can use their average values for the purpose. For the Earth, the solar day (24 h) and the sidereal day (23.56 h) are defined, respectively, as the Sun and a star are taken as reference. They are very close because the Earth's period of rotation is far shorter than its period of revolution around the Sun. In contrast, because Venus has a slow rotation, its solar and sideral days are quite different. Indeed, the canonical value taken for its rotational period (sideral day) is 243.02 d (Konopliv et al. 1999), whereas its solar day varies around 117 d. In the following, we fix the value of the solar venusian day at 117 d to be consistent with the physical inputs in the global circulation model (GCM) simulations. There are mainly two methods of measuring the rotation period and its variations. The radar Doppler measurements give direct access to the instantaneous rotation rate $\omega(t)$ that we can translate into an instantaneous period of rotation LOD(t) by

$$\mathrm{LOD}(t) = \frac{2\pi}{\omega(t)}\cdot \tag{1}$$

Because we are interested in variation around a mean value, we further define $\Delta\omega(t) = \omega(t) - \overline{\omega}$ and $\Delta\mathrm{LOD}(t) = \mathrm{LOD}(t) - \overline{\mathrm{LOD}}$ where $\overline{\mathrm{LOD}} = 243.02$ d and $\overline{\omega} = \frac{2\pi}{\overline{\mathrm{LOD}}}\cdot$ At first order $\frac{\Delta\omega}{\overline{\omega}}(t) = -\frac{\Delta\mathrm{LOD}}{\overline{\mathrm{LOD}}}(t)$. Thanks to infrared images of the surface of the planet, we can also measure the longitude of a reference point ϕ_1 with respect to a given reference system at several epochs where $\dot{\phi}_1 = \omega(t)$. As we discuss in Sect. 7, because $\omega(t)$ has periodic variations, fitting ϕ_1 by a linear function could lead to a wrong estimation of $\bar{\omega}$.

3. Effect of the solid potential on the rotation rate

A first and simplified model to describe the variation in the rotation of Venus is to consider that the atmosphere, mantle, and core of Venus are rotating as a unique solid body. This model enables us to use Kinoshita's theory (1977) with the Andoyer variables (1923) as those describing the rotation of the Earth. The rotation of Venus is then described by three action variables (G, L, H) and their conjugate variables (g, l, h). These G represents the amplitude of the angular momentum, and L and H respectively its projections on the figure axis and on the inertial axis (axis of the reference plane), such as

$$L = G\cos J \quad \text{and} \quad H = G\cos I \tag{2}$$

where I and J correspond respectively to the angles between the angular momentum axis and the inertial axis and between the angular momentum axis and the figure axis. Here the figure axis coincides with the axis of the largest moment of inertia and the inertial axis with the axis of the orbit of Venus at J2000.0. Angle J is still unknown. As shown in Appendix A, for a plausible value of this angle, its impact on the rotation is weaker than the other effects taken into account in this article. For the sake of clarity, J is set to 0 in the following. In this coordinate system, the Hamiltonian related to the rotational motion of Venus is (Cottereau & Souchay 2009)

$$K = F_0 + E + E' + U, \tag{3}$$

where F_0 is the Hamiltonian for the free rotational motion defined by

$$F_0 = \frac{1}{2}\left(\frac{\sin^2 l}{A} + \frac{\cos^2 l}{B}\right)(G^2 - L^2) + \frac{1}{2}\frac{L^2}{C}, \tag{4}$$

where A, B, C are the principal moments of inertia of Venus and E, E' are the components related to the motion of the orbit of Venus, which is caused by planetary perturbations (Kinoshita 1977) and to the choice of the "departure point" as reference point (Cottereau & Souchay 2009). Here U is the disturbing potential of the Sun, considered as a point mass, and is given at first order by

$$U = \frac{\mathcal{G}M'}{r^3}\left[\left[\frac{2C - A - B}{2}\right]P_2(\sin\delta) + \left[\frac{A - B}{4}\right]P_2^2(\sin\delta)\cos 2\alpha\right], \tag{5}$$

where $\mathcal{G}$ is the gravitational constant, M' the mass of de Sun, r the distance between its barycenter and the barycenter of Venus and α and δ are the planetocentric longitude and latitude of the Sun with respect to the mean equator of Venus and a meridian origin (not to be confused with the usual equatorial coordinates defined with respect to the Earth). The P_n^m are the classical Legendre functions given by

$$P_n^m(x) = \frac{(-1)^m\left(1 - x^2\right)^{\frac{m}{2}}}{2^n n!}\frac{\mathrm{d}^{n+m}\left(x^2 - 1\right)^n}{\mathrm{d}x^{n+m}}\cdot \tag{6}$$

The Hamiltonian equations are

$$\frac{\mathrm{d}}{\mathrm{d}t}(L, G, H) = -\frac{\partial K}{\partial(l, g, h)} \tag{7}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(l, g, h) = \frac{\partial K}{\partial(L, G, H)}\cdot \tag{8}$$

Because the components ω_1 and ω_2 of the rotation of Venus are supposed to be negligible with respect to the component ω_3 along the figure axis at first approximation, this yields

$$G = \sqrt{(A\omega_1)^2 + (B\omega_2)^2 + (C\omega_3)^2} \approx C\omega_3 \tag{9}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t}(C\omega_3) \approx \frac{\mathrm{d}}{\mathrm{d}t}(C\omega) \approx -\frac{\partial K}{\partial g}\cdot \tag{10}$$

After splitting ω into its mean value $\overline{\omega}$ and a variation $\Delta\omega$, ω is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}(C\Delta\omega) = -\frac{\partial K}{\partial g}\cdot \tag{11}$$

We notice here that F_0 and $E + E'$ do not contain g, so that the variations in the rotation of solid Venus are only caused by the tidal torque of the Sun included in the disturbing potential U. To make the dependence on the variable g explicit, we express U as a function of the longitude λ and the latitude β of the Sun with respect to the orbit of Venus at the date t with the transformations described by Kinoshita (1977) and based on the Jacobi polynomials, such as

$$\begin{aligned}\frac{\mathrm{d}(C\Delta\omega)}{\mathrm{d}t} &= -\frac{\partial}{\partial g}\frac{\mathcal{G}M'}{r^3}\left[\frac{2C-A-B}{2}\left(-\frac{1}{4}(3\cos^2 I-1)\right.\right.\\ &\quad\left.-\frac{3}{4}\sin^2 I\cos 2(\lambda-h)\right)+\frac{A-B}{4}\\ &\quad\times\left[\frac{3}{2}\sin^2 I\cos(2l+2g)+\sum_{\epsilon=\pm1}\frac{3}{4}(1+\epsilon\cos I)^2\right.\\ &\quad\left.\left.\times\cos 2(\lambda-h-\epsilon l-\epsilon g)\right]\right], \qquad (12)\end{aligned}$$

where β is, by definition, equal to 0 in this case. The first component on the righthand side of Eq. (12) does not depend on the variable g, so that this yields

$$\begin{aligned}\Delta\omega &= -\frac{1}{C}\int\frac{\partial}{\partial g}\left(n^2\left(\frac{a}{r}\right)^3\frac{A-B}{4}\left[\frac{3}{2}\sin^2 I\cos(2l+2g)\right.\right.\\ &\quad+\sum_{\epsilon=\pm1}\frac{3}{4}(1+\epsilon\cos I)^2\\ &\quad\left.\left.\times\cos 2(\lambda-h-\epsilon l-\epsilon g)\right]\right)\mathrm{d}t, \qquad (13)\end{aligned}$$

where $\frac{\mathcal{G}M'}{r^3}$ has been replaced by $n^2\cdot\frac{a^3}{r^3}$, a and n being the semi-major axis and the mean motion of Venus defined by the third Kepler law

$$n^2a^3=\mathcal{G}M'. \qquad (14)$$

Differentiating with respect to g in Eq. (13) gives $\Delta\omega$ as:

$$\begin{aligned}\Delta\omega &= -\int 3\frac{A-B}{4C}n^2\left(\frac{a}{r}\right)^3\left[-\sin^2 I\sin(2l+2g)\right.\\ &\quad+\sum_{\epsilon=\pm1}\frac{1}{2}\epsilon(1+\epsilon\cos I)^2\\ &\quad\left.\times\sin 2(\lambda-h-\epsilon l-\epsilon g)\right]\mathrm{d}t. \qquad (15)\end{aligned}$$

We can write $l+g\approx\phi$ (Kinoshita 1977) where ϕ is the angle between the axis which coincides with the smallest moment of inertia and a reference point arbitrarily chosen as the "departure point" on the orbit of Venus at date t (Cottereau & Souchay 2009). The position of this axis is given with respect to the origin meridian itself defined as the central peak in the crater Adriadne (Konopliv et al. 1999; Davies et al. 1992). In fact ϕ is the angle of proper rotation of the planet ($\dot{\phi}\approx\omega$). In the following, ϕ will be used instead of $l+g$. To solve Eq. (15) analytically, the developments of $(\frac{a}{r})^3\sin 2\phi$ and $(\frac{a}{r})^3\sin 2(\lambda-h\pm\phi)$ are needed as a function of time through the variables M and L_S (the mean anomaly and the mean longitude of Venus, respectively) taking the eccentricity as a small parameter (Kinoshita 1977). In a first approach, the orbit of Venus can be considered as circular (i.e. $e=0$, instead of: $e=0.0068$). This yields

$$\begin{aligned}\Delta\omega &= 3\frac{B-A}{4C}n^2\left[\sin^2 I\ \cos(2\phi)\ \frac{1}{2\dot{\phi}}\right.\\ &\quad-\frac{1}{2}(1+\cos I)^2\cos 2(L_s-\phi)\frac{1}{2\dot{L}_s-2\dot{\phi}}\\ &\quad\left.+\frac{1}{2}(1-\cos I)^2\cos 2(L_s+\phi)\ \frac{1}{2\dot{L}_s+2\dot{\phi}}\right]. \qquad (16)\end{aligned}$$

Table 1. Numerical values, with M_V and R_V the mass and the radius of Venus.

	Venus
Period of revolution	224.70 d (Simon et al. 1994)
Obliquity	2°.6358 (Cottereau & Souchay 2009)
Mean period of rotation	–243.020 d (Konopliv et al. 1999)
Triaxiality: $\frac{A-B}{4C}$	-1.647941×10^{-6} (Konopliv et al. 1999; Yoder 1995)
$\frac{C}{M_V R_V^2}$	0.33600 (Yoder 1995)

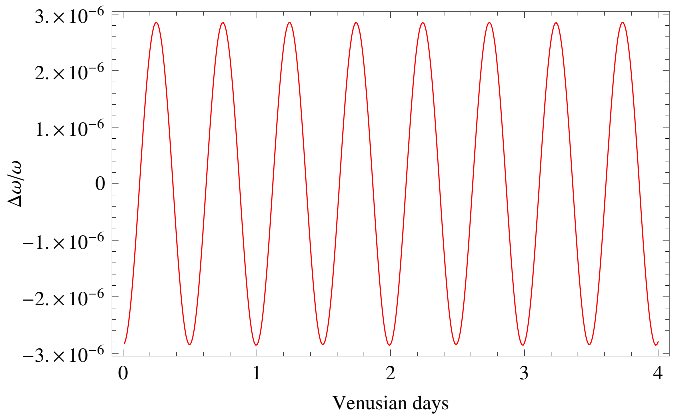

Fig. 1. Variation of $\frac{\Delta\omega}{\omega}$ during 4 venusian solar days (468 d). One venusian solar day corresponds to 117 d.

This equation is similar to the equation given by Woolard (1953) in his theory of the Earth's rotation, using a different formalism based on classical Euler angles.

With the numerical values of Table 1, the largest terms of the variation of $\frac{\Delta\omega}{\omega}$ are

$$\begin{aligned}\frac{\Delta\omega}{\omega} &= 2.77\times10^{-6}\cos(2L_S-2\phi)\\ &\quad+6.12\times10^{-9}\cos 2\phi\\ &\quad-1.99\times10^{-11}\cos(2L_S+2\phi) \qquad (17)\end{aligned}$$

where $2L_S-2\phi$, 2ϕ, and $2L_S+2\phi$ correspond to the leading terms with respective periods 58 d, 121.80 d, and 1490 d. As the two last terms scale as the square of the obliquity, they are significantly smaller than the term with argument $2L_S-2\phi$ (2.77×10^{-6}). As Venus has a very slow rotation that appears in the scaling factor $3\frac{B-A}{4C}\frac{n^2}{\omega}$, the variations in the rotation rate due to the solid torque are greater than the Earth's ones, which correspond to amplitudes of $\frac{\Delta\omega}{\omega}\approx10^{-10}$.

Figure 1 shows the relative variations in the speed of rotation of Venus due to the solid torque exerted by the Sun during a four venusian day time span (468 d). The orientation of the bulge of Venus at $t=0$ is given with respect to the origin meridian. The mean orbital elements are given by Simon et al. (1994). The choice of the time span will allow us to compare the different effects that act on the rotation of Venus.

The physical meaning of the leading term with argument $2L_S-2\phi$ can be understood using a simple toy model. Consider a coplanar and circular orbit. At quadrupolar order, the gravitational potential created by the planet in its equatorial plane is equal to the one created by three point masses, one located at the center of the planet and of mass $M_v-2\mu$ and two other ones symmetrically located on the surface of the planet along the axis of smallest moment of inertia and of mass μ (see Fig. 2), where

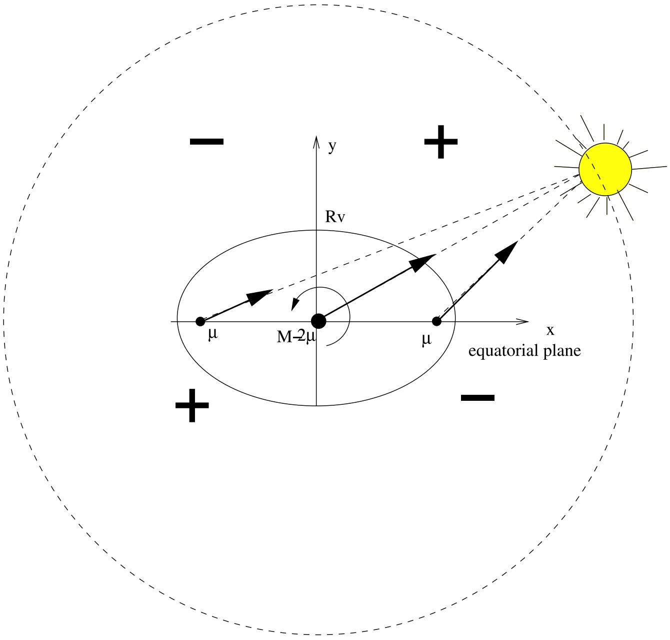

Fig. 2. Diagram of the gravitational force exerted by the Sun in the frame corotating with Venus. The − and + signs give the sign of the net torque in each quadrant.

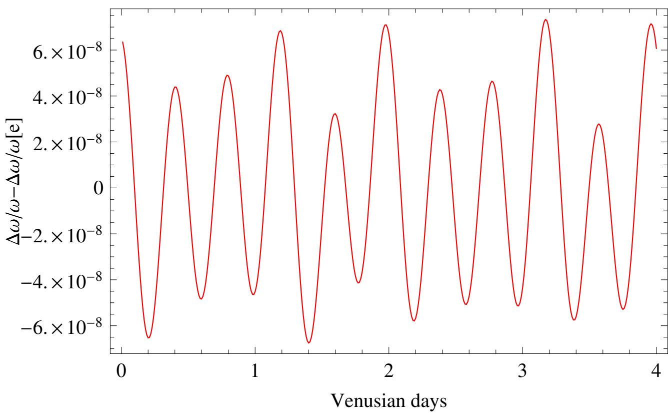

Fig. 3. Influence of the eccentricity in the variation of the speed of rotation of Venus.

$\mu = \frac{B-A}{2R_V^2}$, R_V is the planet mean radius. The torque exerted on the planet by the Sun can thus be computed using the forces exerted only on these three points. As shown in Fig. 2, the sign of the resulting torque depends on the quadrant of the x, y plane the Sun is located in, and thus changes four times during a solar day, whose the corresponding argument is $2L_S - 2\phi$.

To evaluate the influence of the eccentricity of Venus on its rotation rate, we show in Fig. 3 the residuals after substraction of $\frac{\Delta\omega}{\overline{\omega}}$ when the eccentricity is taken into account in the development of $(\frac{a}{r})^3 \sin 2\phi$ and $(\frac{a}{r})^3 \sin 2(\lambda - h \pm \phi)$ with respect to the simplified expression given by Eq. (17). We can observe that the eccentricity of Venus acts on $\frac{\Delta\omega}{\overline{\omega}}$ with an amplitude of $\approx 10^{-8}$ for the terms $2L_s + M - 2\phi$ (46.34 d) and $2L_s - M - 2\phi$ (78.86 d). These variations are smaller than the leading coefficient seen in Eq. (17) by two orders and greater than the other coefficients with arguments 2ϕ and $2\phi + 2L_s$.

From the relation $\frac{\Delta\omega}{\omega}(t) = -\frac{\Delta \mathrm{LOD}}{\mathrm{LOD}}(t)$, the variations in the LOD(t) due to the torque of the Sun are 120 s (i.e. 2 mn or 0.0014 d). The implication of these results is discussed in Sect. 7. These variations are larger than the uncertainties on the period of rotation measurements of Venus given by Magellan, that is to say, 243.0200 ± 0.0002 d in Konopliv et al. (1999) or 243.0185 ± 0.0001 d in Davies et al. (1992), and close to the amount of their difference.

4. Effects of the elasticity on the rotation rate: deformation due to the zonal tidal potential

For the Earth, a large variation of the speed of rotation comes from to the zonal potential, which causes temporal variations in the moment of inertia C. In this section, these zonal effects are evaluated for Venus, which is considered as a deformable body. The zonal part of the potential exerted by the Sun on a point M at the surface of the planet is given by the classical formula

$$V_{2,S}(\delta_M, \delta) = \frac{9}{4}\mathcal{G}M'\frac{R_V^2}{r^3}\left(\sin^2\delta - \frac{1}{3}\right) \times \left(\sin^2\delta_M - \frac{1}{3}\right), \quad (18)$$

where δ and δ_M represent the planetocentric latitude of the Sun (the disturbing body) and of the point M, and r the distance between the barycenter of the Sun and the Venus one. The corresponding bulge produced has a potential (Melchior 1978):

$$\Delta V_{2,V}(\delta_M, \delta, r') = k_2 V_{2,S}\frac{R_V^3}{r'^3}, \quad (19)$$

where k_2 and R_V are the Love number and the radius of the planet. Differentiating the potential produced by Venus at its surface (MacCullagh's formula), $\Delta V_{2,V}$ can be expressed as a function of the principal moments of inertia

$$\Delta V_{2,V}(\delta, r') = \frac{3}{2}\frac{\mathcal{G}}{r'^3}(\mathrm{d}C - \mathrm{d}A)\left(\sin^2\delta - \frac{1}{3}\right), \quad (20)$$

where we take $\mathrm{d}A = \mathrm{d}B$ as the deformation is purely zonal. By identification we obtain

$$k_2\frac{3}{4}\frac{M'}{M_V}\left(\sin^2\delta - \frac{1}{3}\right)\left(\frac{R_V}{r}\right)^3 = \frac{\mathrm{d}C - \mathrm{d}A}{2}\frac{1}{M_V R_V^2}. \quad (21)$$

Because this deformation does not induce a change in the volume of the isodensity surfaces in the planet to first order, $\mathrm{d}I = \frac{1}{2}\mathrm{d}C + \mathrm{d}A = \frac{1}{2}\Delta C + \Delta A = 0$ (see Melchior 1978, for a rigorous demonstration). Euler's third equation is $C\omega$ = const., and to first order this yields $\frac{\Delta C}{C} = -\frac{\Delta\omega}{\overline{\omega}}$. Thus the variations in the angular rate of rotation of Venus are given by

$$\frac{\Delta\omega}{\overline{\omega}} = -\frac{\Delta C}{C} = -k_2\left(\frac{R_V}{a}\right)^3\left(\frac{a}{r}\right)^3\frac{M'}{M_V}\left(\sin^2\delta - \frac{1}{3}\right)\frac{M_V R_V^2}{C} \quad (22)$$

where M_V is the mass of Venus. Expressing $\sin^2\delta_1$ as a function of I and λ where I and λ are given in Sect. 3 and represent the

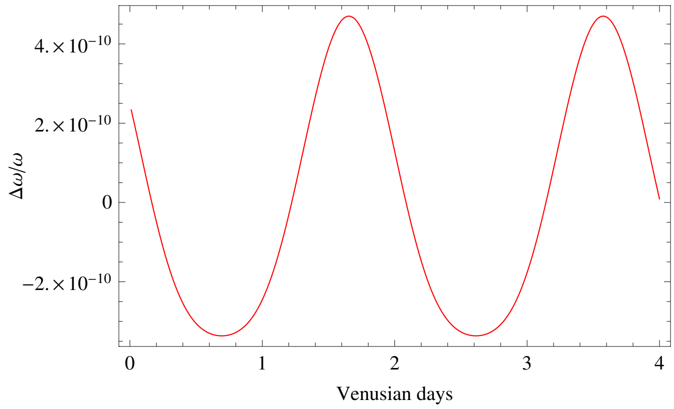

Fig. 4. $\frac{\Delta\omega}{\overline{\omega}}$ due to the zonal potential during 4 venusian days (468 d).

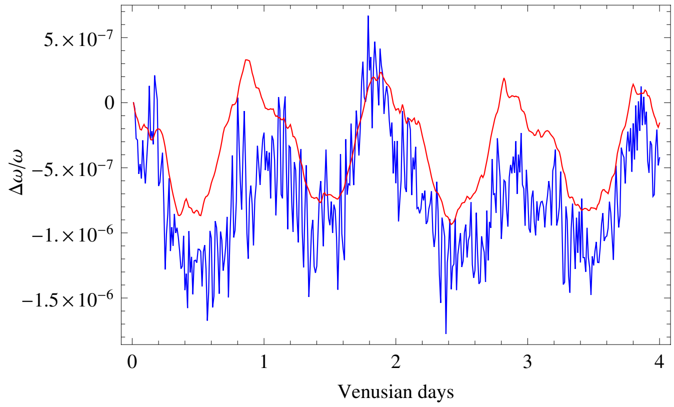

Fig. 5. $\frac{\Delta\omega}{\overline{\omega}}$ due to the atmosphere during 4 venusian days for the GCM1 (red curve) and the GCM2 (blue curve).

obliquity of Venus and the true longitude of the Sun, Eq. (22) becomes

$$\frac{\Delta\omega}{\overline{\omega}} = -k_2\frac{M_V R_V^2}{C}\frac{M'}{M_V}\left(\frac{R_V}{a}\right)^3\left(\frac{a}{r}\right)^3 \times \left(\frac{\sin^2 I}{2}(1-\cos 2\lambda) - \frac{1}{3}\right). \tag{23}$$

Based on the developments of $(\frac{a}{r})^3$ and $(\frac{a}{r})^3 \cos 2\lambda$ with respect to time from Cottereau & Souchay (2009), Fig. 4 shows the relative variation in the speed of rotation of Venus due to the zonal potential. The numerical values of $k_2 = 0.295 \pm 0.066$ and the ratio $\frac{C}{M_V R_V^2} = 0.3360$ are taken from Konopliv & Yoder (1996) & Yoder (1995) respectively. The expressions found correspond to variations in the LOD(t) of 0.019 s peak to peak, which is much smaller than the contributions studied in Sect. 3. The zonal part of the potential on a nonrigid Venus thus has little influence on its very slow rotation, in contrast to the Earth, for which these variations with annual and semi-annual components are not negligible (on the order of 10^{-3} s) with respect to the 1d rotation (Yoder et al. 1981; Souchay & Folgueira 1998). The semi-annual components are particularly small for Venus because its eccentricity and obliquity are much lower than their respective value for the Earth.

5. Atmospheric effects on the rotation of Venus

As Venus has a denser atmosphere than the Earth, and as we now know that the Earth's atmosphere acts on its rotation in a significant manner (Lambeck 1980), it will be interesting to study the corresponding effects on Venus. In this section, the core and the mantle of Venus are supposed to be rigidly coupled, and its atmosphere rotates at a different rate. The variations of the speed of rotation in Venus due to its atmosphere are given by

$$\frac{\Delta G}{G} = \frac{\Delta\omega}{\overline{\omega}}, \tag{24}$$

where G and ΔG represent the angular momentum of the rigid Venus and its variation due to the atmosphere $\Delta G = -\Delta G_{\mathrm{atm}}$. The angular momentum ΔG_{atm} of the atmosphere can be split into two components:

- the matter term G_M, which is the product of ω with the inertia momentum of the atmosphere;
- the current term G_w, which is due to the wind motions with respect to the frame that is solidly rotating with the planet.

From Lebonnois et al. (2010a) we have

$$\begin{aligned} G_{\mathrm{atm}} &= (1+k_2')G_M + G_w \\ &= (1+k_2')\frac{\omega R_V^4}{g}\int\int_s P_s \cos^3\theta \mathrm{d}\theta\mathrm{d}\phi \\ &\quad + \frac{R_V^3}{g}\int\int\int_v \cos^2\theta v_\theta \mathrm{d}h\mathrm{d}\theta\mathrm{d}\phi, \end{aligned} \tag{25}$$

where θ, ϕ and R_V stand respectively for the latitude, longitude, and radius of Venus, v_θ is the zonal wind, and k_2' the load Love number of degree 2 (Karatekin et al. 2010). To determine the variations in the angular momentum, two simulations made with the global circulation model (GCM) of the Laboratoire de Meteorologie Dynamique (LMD) (Lebonnois et al. 2010b) are used and compared. These simulations were obtained with the LMD Venus general circulation model, using conditions similar to those presented in Lebonnois et al. (2010a), except for the boundary layer scheme. The first simulation (GCM1) was integrated from a zero wind state and is very close to the simulation published in Lebonnois et al. (2010a), though the winds in the deep atmosphere (0 to 40 km altitude) are slightly higher, owing to the updated boundary layer scheme. The second one (GCM2) was integrated with initial winds in super rotation where the resulting winds in the deep atmosphere are close to observed values. The results presented in Fig. 5 were obtained after a 200 day integration for GCM1 and 130 days for GCM2. Here we calculate the seasonal variation of the angular velocity of Venus induced by the atmospheric winds to compare it with the variation discussed in Sect. 3. The numerical values of the radius and the principal moment of inertia are taken in Cottereau & Souchay (2009).

Figure 5 shows the relative variation in the speed of rotation of Venus due to the atmosphere for 4 venusian days (470 d) with the GCM1 (red curve) and GCM2 (blue curve). These variations have an amplitude of 1.26×10^{-6} peak to peak for GCM1 and 2.44×10^{-6} for GCM2. For the two models, the large amplitude of $\frac{\Delta\omega}{\overline{\omega}}$ of the order of 10^{-6} comes from the current term and depends on winds of Venus whereas the matter term G_M acts on the rotation with an amplitude on the order of 10^{-10}.

Using the definition of the LOD(t) given in Sect. 2, the atmospheric contributions to Venus rotational speed correspond to peak to peak variations on the LOD(t) of 27 s with the GCM1 and 51 s with the GCM2. These values are consistent with the value of LOD(t) for the planet of 7.9 s given by Karatekin et al. (2009, 2010), who used the simulation presented in Lebonnois et al. (2010a). The differences are related to the amplitudes of the zonal winds in the region of maximum angular momentum (10–30 km of altitude), which vary between the different simulations. The most realistic values for these winds are obtained with the GCM2 simulation, where the winds are close to the observational data obtained from the Venera and Pioneer Venus missions (Schubert 1983). We compared here the results given by GCM1 and GCM2 to give an idea of the uncertainties in our present understanding of the atmospheric circulation and its modeling. In the following, only the values obtained with the model GCM2 are compared to the other effects because it is thought to be the most realistic. The atmospheric effects are nearly two times less than the solid effects described in Sect. 3.

6. Core effects on the rotation of Venus

The interior of Venus is probably liquid as inferred from the orbiting spacecraft data (Konopliv & Yoder 1996). The internal properties of Venus are expected to be like the Earth with a core radius around 3120 km (Yoder 1995), but with a noteworthy difference because there is no dynamo effect on Venus (Nimmo 2002). In this section, we investigate the impact of such a core on the rotational motion of Venus and especially on the variation of its LOD(t). For that purpose, we numerically integrate the rotational motion of Venus by taking the inertial pressure torque into account. The equations governing the rotational motion of two-layer Venus are the angular momentum balance for the whole body

$$\frac{\mathrm{d}\boldsymbol{H}}{\mathrm{d}t} + \boldsymbol{\omega} \wedge \boldsymbol{H} = \boldsymbol{\Gamma} \tag{26}$$

and for the core

$$\frac{\mathrm{d}\boldsymbol{H}_\mathrm{c}}{\mathrm{d}t} - \boldsymbol{\omega}_\mathrm{c} \wedge \boldsymbol{H}_\mathrm{c} = 0, \tag{27}$$

(Moritz & Mueller 1987) where $\boldsymbol{H}, \boldsymbol{H}_\mathrm{c}$ are respectively the angular momentum of Venus and of the core (see also e.g. Rambaux et al. 2007). The vector Γ is the gravitational torque acting on Venus. Here, we assume that the core has a simple motion as suggested by Poincaré in 1910, and we neglect the core-mantle friction arising at the core-mantle boundary. Then, the rotational motion of each layer of Venus is integrated simultaneously with the gravitational torque due to the Sun acting on the triaxial figure of Venus. The orbital ephemerides DE421 (Folkner et al. 2008) are used for the purpose.

First we double-checked the good agreement between the numerical and the analytical solutions given by Eq. (17) for a simplified model of a one-layer, solid, rigid Venus case. The very small differences obtained (at the level of a relative 10^{-10}) may result from using of different ephemerides (DE421 for the numerical approach and VSOP87 for the analytical one). Then we applied the procedure for the two-layer case. As the moments of inertia I_c of the core are not yet constrained by measurements, we used internal models (Yoder 1995), for which I_c is taken from 0.01 to 0.05, according to the size of the core. The flattening of the core is scaled to the flattening of the mantle by assuming that the distribution of mass anomalies is the same.

Table 2. Resulting amplitude for Venus $\frac{\Delta\omega}{\omega}$ with a fluid core (expressed in unit of 10^{-6}), with last line computed for a model without a fluid core.

Period / I_c	58.37 d $2L_\mathrm{s} - 2\phi$	46.34 d $2L_\mathrm{s} - 2\phi + M$	78.86 d $2L_\mathrm{s} - 2\phi - M$	121.51 d 2ϕ
0.01	2.859979	0.053456	0.013032	0.006328
0.02	2.950486	0.055147	0.013445	0.006528
0.03	3.046908	0.056950	0.013884	0.006741
0.04	3.149846	0.058874	0.014353	0.006968
0.05	3.259981	0.060932	0.014855	0.007211
–	2.774866	0.052244	0.012726	0.006120

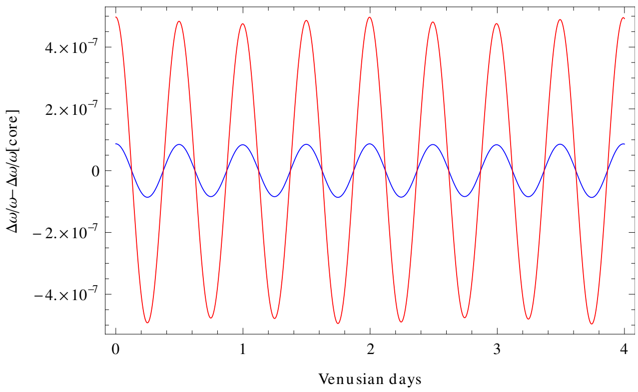

Fig. 6. $\frac{\Delta\omega}{\omega}$ due to the core during 4 venusian days with a $I_\mathrm{c} = 0.05$ (red curve) and a $I_\mathrm{c} = 0.01$ (blue curve).

Table 2 shows the new amplitudes of $\frac{\Delta\omega}{\omega}$ for the four main oscillations with arguments $2L_\mathrm{s} - 2\phi$, $2L_\mathrm{s} - 2\phi + M$, $2L_\mathrm{s} - 2\phi - M$, 2ϕ and with corresponding periods 58.37 d, 46.34 d, 78.86 d and 121.51 d. The presence of the fluid core allows a differential rotation of the mantle and of the interior of the planet. As a consequence in first approximation the mantle is decoupled from the interior and presents a larger amplitude of libration than in the solid case. The amplitude of the libration increased with the size of the core. At maximum, for $I_\mathrm{c} = 0.05$, we obtain an increase in the oscillation of $2L_S - 2\phi$ of 17% corresponding to a variation of +20.4 s.

Figure 6 shows the relative variation in the speed of rotation of Venus due to the core during 4 venusian days with $I_\mathrm{c} = 0.05$ and $I_\mathrm{c} = 0.01$. These variations have peak-to-peak amplitudes of 1.7×10^{-7} and 9.7×10^{-7}. Using the definition of the LOD(t) given in Sect. 2, the core contributions to Venus' rotational speed correspond to peak-to-peak variations on the LOD(t) between 3.6 s and 20.4 s. Although the core effects on the rotation of Venus increase with the core size, they are smaller than the solid and atmospheric effects. Indeed, when $I_\mathrm{c} = 0.05$, the core effects are nearly two and six times less than the atmospheric and solid ones.

7. Comparison and implication for observations

The variations in the rotation of Venus presented in this paper are quasi-periodic, and they are mainly due to three kinds of effects: solid, atmospheric, and core. Compared to them, the zonal potential has a negligible influence. Finally $\omega(t)$ can be written

Table 3. Variation of $\frac{\Delta\omega}{\bar{\omega}}$ due to the solid and core effects.

Period P_i $\frac{2\pi}{\omega_i}$	Argument	Solid $a_{s,i}$ $\cos\omega_i t$	Core $a_{c,i}$ $\cos\omega_i t$
58 d	$2L_s - 2\phi$	2.77×10^{-6}	$8.51 \times 10^{-8} < a_c < 4.85 \times 10^{-7}$
46.34 d	$2L_s + M - 2\phi$	5.24×10^{-8}	$1.01 \times 10^{-9} < a_c < 8.5 \times 10^{-9}$
78.86 d	$2L_s - M - 2\phi$	1.27×10^{-8}	$3.1 \times 10^{-10} < a_c < 2.1 \times 10^{-9}$
121.80 d	2ϕ	6.12×10^{-9}	$2.1 \times 10^{-10} < a_c < 1.1 \times 10^{-9}$

in the form

$$\begin{aligned} \omega(t) &= \bar{\omega} + \Delta\omega \\ &= \bar{\omega} + \left[\sum_{\mathrm{i}} (\mathrm{a}_{\mathrm{s},i} + a_{\mathrm{c},i}) \cos(\omega_i t + \rho_i) \right. \\ &\quad \left. + \sum_j a_{\mathrm{a},j} \cos(\omega_j t + \rho_j) \right] \bar{\omega}, \end{aligned} \tag{28}$$

where ω_i, ρ_i and $a_{\mathrm{s},i}, a_{\mathrm{c},i}$ (see Table 3) are the frequencies, the phases, and the corresponding amplitudes of the variation of rotation of Venus due to the solid and the core, and ω_j, $a_{\mathrm{a},j}$ (see Table 4) for those due to the atmosphere. The atmospheric coefficients as well as their periods have been obtained from a fast Fourier transform (FFT) where ρ_j are the phases. The power spectrum of the atmospheric variation is complex and can vary significantly from one model to another. As a consequence, only the most important frequencies are shown for the GCM2 model in Table 4.

Comparing the amplitudes given in Tables 3 and 4, we see that the greater effect on the rotation rate of Venus is caused by the solid potential exerted by the Sun on its rigid body. If all effects are taken into account, the variations in the LOD(t) can reach 3 mn, which could be observable in the future.

As most past studies used infrared imaging of the surface to measure the evolution in time of the longitude of reference points ϕ_1 at Venus surface, let us consider how this variable behaves in our model. As $\dot{\phi}_1 \approx \omega(t)$ we have

$$\phi_1(t) = \phi_{1,0} + \overline{\omega}t + \int_{t_0}^{t} \Delta\omega \mathrm{d}t. \tag{29}$$

Figure 7 shows the variations $\Delta\phi_1$ during four venusian days, which are caused by the combined effects of the solid, the core, and the atmosphere modeled by the GCM2.

Like the variations in the rotation rate, these variations are periodic with a peak-to-peak amplitude reaching 12 m (time differences are converted into distance at the surface). In preceding studies that measured the mean rotation rate of Venus, these variations were neglected. This implied fitting the measurements of the phase angle ϕ_1 with a line of constant slope, this slope giving the mean rotation rate. Because the variations in the rotation rate discussed above are not negligible, we show in the following that this approach can yield large errors in the derived value of $\bar{\omega}$, especially for a short interval of time. Similar to Laskar & Simon (1988) let us consider the error made on the mean rotation rate when fitting the signal given by Eq. (29), keeping only the most important frequency for simplicity, by a function of the type

$$\phi_1(t) = \phi_{1,\mathrm{obs}} + \omega_{\mathrm{obs}}\, t \tag{30}$$

over a time span $[t_0, t_0 + T]$. For a least square fitting procedure, the residual is given by

$$D = \int_{t_0}^{t_0+T} (\phi_{1,0} + \bar{\omega}t + A \sin\omega_1 t - (\omega_{\mathrm{obs}}\, t + \phi_{1,\mathrm{obs}}))^2 \mathrm{d}t \tag{31}$$

Table 4. Variations of $\frac{\Delta\omega}{\bar{\omega}}$ due to the atmosphere effects modeled by GCM2.

Period P_j $\frac{2\Pi}{\omega_j}$	$a_{a,j}$ $\cos\omega_j t + \rho_j$	ρ_j
117 d	4.17×10^{-7}	3.11
266 d	3.22×10^{-7}	1.74
5.02 d	1.74×10^{-7}	2.39
6.09 d	1.41×10^{-7}	3.14
40.6 d	1.28×10^{-7}	0.0061
79.5 d	1.21×10^{-7}	1.64

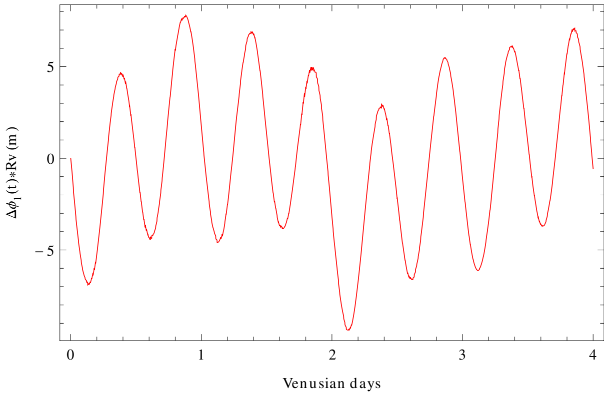

Fig. 7. Variations of $\Delta\phi_1 \times R_V$ in meter during a four venusian days time span.

where A and $\omega_1 = \frac{2\pi}{58}$ rd/d correspond respectively to the larger amplitude of Eq. (29) and its corresponding frequency. Minimizing this residual yields

$$\begin{aligned} \omega_{\mathrm{obs}} &= \bar{\omega} - \frac{6A}{T^3\omega_1^2} \Big[T\omega_1 \big[\cos(\omega_1 t_0) + \cos\omega_1(T + t_0)\big] \\ &\quad + 2\big[\sin(\omega_1 t_0) - \sin\omega_1(T + t_0)\big] \Big] \end{aligned} \tag{32}$$

$$\begin{aligned} \phi_{1,\mathrm{obs}} &= \phi_{1,0} + \frac{2A}{T^3\omega_1^2} \Big[T(2T + 3t_0)\omega_1 \cos(\omega_1 t_0) \\ &\quad + T(T + 3t_0)\omega_1 \cos\omega_1(T + t_0) \\ &\quad + 3(T + 2t_0)\big[\sin(\omega_1 t_0) - \sin\omega_1(T + t_0)\big] \Big]. \end{aligned} \tag{33}$$

We can see that ω_{obs} depends on both the time of the first observation (t_0) (i.e. phase) and interval (T) between observations. If the phase of the effect is unknown, the max error made on the mean rotation rate is given by

$$\Delta\mathrm{LOD}_{\mathrm{obs}} = \max_{t_0} |\omega_{\mathrm{obs}}(t_0, T) - \bar{\omega}| \frac{2\pi}{\bar{\omega}^2}. \tag{34}$$

Figure 8 shows this maximum error $\Delta\mathrm{LOD}_{\mathrm{obs}}$ in seconds as a function of the interval T between observations.

As we can see, even if the mean angular velocity is retrieved for long baseline observations, the error yielded by the linear fit can be large for observations of short duration. In addition, the use of such a simple model for ϕ_1 prevents any measurement of the amplitude of the variations detailed in this paper. To measure the amplitude of these variations, modeling ω and ϕ_1 with

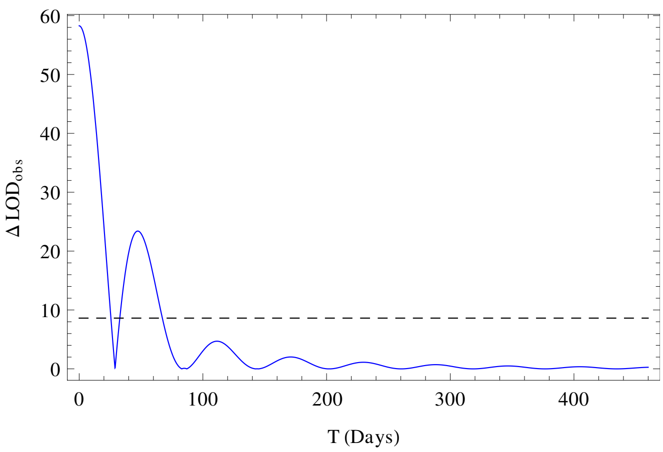

Fig. 8. Maximum error of $\Delta LOD_{obs}(T)$ in seconds given by Eq. (34). The dotted line is the uncertainty given by Magellan (Davies 1992) on the rotation of Venus.

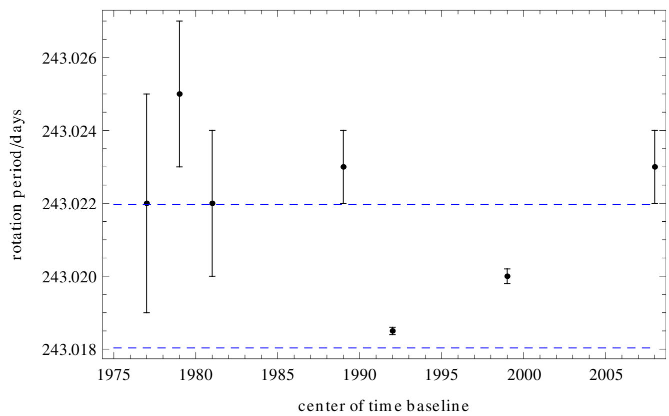

Fig. 9. Values of the period of rotation of Venus and their error bars given since 1975. The dotted lines represent variations of 0.00197 d around the mean values of 243.020 d caused by the effects presented in this paper.

Eqs. (28) and (29), respectively, during the data reduction is necessary.

As the greater effect is due to the torque of the Sun on the Venus rigid body with a large amplitude on a 58 d interval, it could be interesting to subtract the measured signal by a fitted sinusoid of this frequency. The direct measurement of the amplitude of the sinusoid would give information on the triaxiality of Venus at 3 to 17% of error because of the core contribution.

To disentangle atmospheric effects, a multifrequency analysis of the data will be necessary. Indeed, after having determined the larger amplitude on 58 d as explained previously, it could be subtracted from the analysis. Then as the atmospheric winds (described by Eq. (25)) are the second most important effects on the rotation of Venus, the residuals obtained could constrain their strength. In parallel it should also be interesting to fit the signal obtained by a sinusoid on the period of 117 d because only the atmosphere acts on the rotation with this periodicity. This period is also present when using an alternative atmospheric model GCM1 (Lebonnois 2010a), so the corresponding amplitude could directly give indications on the atmospheric winds. Fitting the signal with additional sinusoids with other periods given in Table 4 could also increase the constraints.

At last, as the core does not make the same contribution on each period presented in Table 3, fitted the signal by Eq. (28) (or by Eq. (29)) could confirm the presence of a fluid core. Of course, such a detection is only possible if the precision of the measurements is of the same order of magnitude as the core effects and if the atmospheric effects that add noise in the signal are better modeled in the future.

Many values of the mean rotation of Venus have been estimated since 1975. Figure 9 shows all these values, as well as their error bars. The latter value of 243.023 ± 0.001 d has been recently estimated from Venus express VIRTIS images (Mueller et al. 2010, submitted). The dotted lines represent variations of 0.00197 d around a mean value of 243.020 ± 0.0002 d (Konopliv et al. 1999) caused by solid, atmospheric (GCM2), and core (I_c = 0.05) effects presented in this paper. The value of Davies et al. (1992) set to 243.0185 ± 0.0001 d has been recommended by the IAU (Seidelmann et al. 2002).

As we can see from Fig. 9:

- the variations of the period in rotation (0.00197 d) presented in this paper are consistent with most of the mean rotation periods measured so far;
- if the true mean value of the period of rotation of Venus is close to the IAU value, the variations cannot explain the most recent value obtained by VIRTIS (Mueller et al. 2010, submitted). Indeed, the difference between them of 7 mn implies larger variations.

The different values of the rotation of Venus since 1975 could be explained by the variations of 0.00197 d due to the solid, atmospheric, and core effects. Despite the value of Davies et al. (1992) having recommended by the IAU because of its small published error bars (±0.0001), its large difference with the recent VIRTIS value (7 mn) and the other measurements, does not agree with the variations presented in this paper. It seems difficult to explain these large variations, which would imply an increase of more than 50% of the effects discussed here, with the current models, so it would be interesting to compare in detail the different rotation rate measurement methods and to determine with a better accuracy the mean value of the rotation of Venus.

8. Conclusion

The purpose of this paper was to detemine and to compare the major physical processes influencing the angular speed of rotation of Venus. We Applied different theories already used for the Earth to evaluate the variation of the rotation rate as well as of the LOD.

We computed the effect of the solid potential exerted by the Sun on a rigid Venus using the theory derived from Kinoshita (1977). Considering in the first step that the orbit of Venus is circular, we found that the variations in the rotation rate have a large amplitude of 2.77×10^{-6} with argument $2L_s - 2\phi$ (58 d). Taking the eccentricity of the orbit into account adds periodic variations with a 10^{-8} amplitude. Unlike Earth, because Venus has a very slow rotation, the solid potential has a leading influence on the rotation rate that corresponds to peak-to-peak variations in the LOD of 120 s

Considering Venus as an elastic body, we then evaluated the impact of the zonal tidal potential of the Sun. These variations correspond to peak-to-peak variations in the LOD of 0.014 s, which are very small with respect to the contributions of the solid effect.

Then we computed the effects of both the core and the atmosphere (modeled by two different simulations of the LMD global circulation model). According to our computation the atmospheric and core contributions to Venus rotational speed correspond to peak-to-peak variations of the LOD of 25–50 s and 3.5–20.4 s, respectively, at different periods. Despite its thickness, the impact of the venusian atmosphere modeled by our most realistic simulation on the rotation is 2.4 times less than the contribution of the solid torque exerted by the Sun. The variations in the LOD due to the core, which increase with its size, are still smaller.

Finally we have shown that the variations of ω and $R_V\phi_1$, which reach 3 mn and 12 m, need to be taken into account when reducting the observations. Ignoring these variations could lead to an incorrect estimation of $\bar{\omega}$. With the steadily increasing precision of the measurements, carrying a frequency analysis of the data modeled by either Eqs. (28) or (29) will hopefully put physical constraints on the physical properties of Venus (triaxiality, atmosphere, core).

To conclude, the variations shown in this paper would explain different values of the mean rotation of Venus given since 1975. The difference of 7 mn between the IAU value (243.0185, Davies et al. 1992) and the VIRTIS one (243.023 ± 0.001, Mueller et al. 2010, submitted) implies larger variations, not found here, probably due to systematic errors in the measurements of the rotation. In view of these new results and the recent study of Mueller et al. (2010, submitted), it would be interesting to compare the different methods in measuring the rotation of Venus, so as to identify the error sources with better accuracy and to revise the value of the mean rotation.

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Appendix A: Influence of the angle *J* on the rotation rate

In Sect. 3, we assumed that the angle J between the angular momentum axis and the figure axis of Venus can be neglected. Here we reject this hypothesis of the coincidence of the poles and we evaluate the impact of J on the rotation. According to Eq. (5) the variations of $\Delta\omega$ is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}(C\Delta\omega) = -\frac{\partial U}{\partial g}, \tag{A.1}$$

where

$$U = \frac{\mathcal{G}M'}{r^3}\left[\left[\frac{2C-A-B}{2}\right]P_2(\sin\delta) + \left[\frac{A-B}{4}\right]P_2^2(\sin\delta)\cos 2\alpha\right], \tag{A.2}$$

As in Sect. 3, we express $P_2(\sin\delta)$ and $P_2^2(\sin\delta)\cos 2\alpha$ as functions of the longitude λ and the latitude β of the Sun with the transformation described by Kinoshita (1977) without supposing $J = 0$, such as

$$\begin{aligned}
P_2(\sin\delta) = {} & \frac{1}{2}(3\cos^2 J - 1)\Bigg[\frac{1}{2}(3\cos^2 I - 1)P_2(\sin\beta) \\
& - \frac{1}{2}\sin 2I\sin(\lambda - h)P_1^2(\sin\beta) \\
& - \frac{1}{4}\sin 2IP_2^1(\sin\beta)\cos 2(\lambda - h)\Bigg] \\
& + \sin 2J\Bigg[-\frac{3}{4}\sin 2IP_2(\sin\beta)\cos g \\
& - \sum_{\epsilon=\pm1}\frac{1}{4}(1+\epsilon\cos I)(-1+2\epsilon\cos I) \\
& \times P_2^1(\sin\beta)\sin(\lambda - h - \epsilon g) \\
& - \sum_{\epsilon=\pm1}\frac{1}{8}\epsilon\sin I(1+\epsilon\cos I) \\
& \times P_2^2(\sin\beta)\cos(2\lambda - 2h - \epsilon g)\Bigg] \\
& + \sin^2 J\Bigg[\frac{3}{4}\sin^2 IP_2(\sin\beta)\cos 2g + \frac{1}{4}\sum_{\epsilon=\pm1}\epsilon\sin I \\
& \times (1+\epsilon\cos I)P_2^1(\sin\beta)\sin(\lambda - h - 2\epsilon g) - \frac{1}{16} \\
& \times \sum_{\epsilon=\pm1}(1+\epsilon\cos I)^2 P_2^2(\sin\beta)\cos 2(\lambda - h - \epsilon g)\Bigg]
\end{aligned} \tag{A.3}$$

and

$$\begin{aligned}
P_2^2(\sin\delta)\cos 2\alpha = {} & 3\sin^2 J\Bigg[-\frac{1}{2}(3\cos^2 I - 1)P_2(\sin\beta)\cos 2l \\
& + \frac{1}{4}\sum_{\epsilon=\pm1}\sin 2IP_2^1(\sin\beta)\sin(\lambda - h - 2\epsilon l) \\
& + \frac{1}{8}\sin^2 IP_2^2(\sin\beta)\cos 2(\lambda - h - \epsilon l)\Bigg] \\
& + \sum_{\rho=\pm1}\rho\sin J(1+\rho\cos J) \\
& \times \Bigg[-\frac{3}{2}\sin 2IP_2(\sin\beta)\cos(2\rho l + g) \\
& - \sum_{\epsilon=\pm1}\frac{1}{2}(1+\epsilon\cos I)(-1+2\epsilon\cos I) \\
& \times P_2^1(\sin\beta)\sin(\lambda - h - 2\rho\epsilon l - \epsilon g) \\
& - \sum_{\epsilon=\pm1}\frac{1}{4}\epsilon\sin I(1+\epsilon\cos I) \\
& \times P_2^2(\sin\beta)\cos(2\lambda - 2h - 2\rho\epsilon l - \epsilon g)\Bigg] \\
& + \sum_{\rho=\pm1}\frac{1}{4}(1+\rho\cos J)^2\Bigg[-3\sin^2 IP_2(\sin\beta) \\
& \times \cos(2l + 2\rho g)\sum_{\epsilon=\pm1}\epsilon\sin I(1+\epsilon\cos I) \\
& \times P_2^1(\sin\beta)\sin(\lambda - h - 2\rho\epsilon l - 2\epsilon g) \\
& + \sum_{\epsilon=\pm1}\frac{1}{4}(1+\epsilon\cos I)^2 \\
& \times P_2^2(\sin\beta)\cos 2(\lambda - h - \rho\epsilon l - \epsilon g)\Bigg].
\end{aligned} \tag{A.4}$$

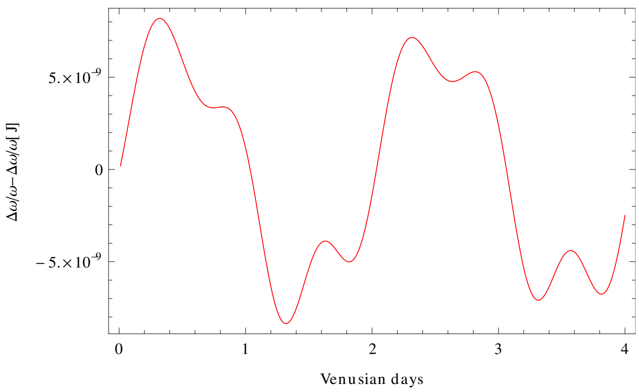

Fig. A.1. Influence of the angle $J = 0.5°$ in the variation of the speed of rotation of Venus.

Assuming that $\beta = 0$ and removing the components which do not depend on the variable g this yields:

$$\begin{aligned}\frac{\mathrm{d}(C\Delta\omega)}{\mathrm{d}t} &= -\frac{\partial}{\partial g}\frac{\mathcal{G}M'}{a^3}\left(\frac{a}{r}\right)^3\Bigg[\frac{2C-A-B}{2}\\
&\times\Bigg(\sin 2J\Bigg[\frac{3}{4}\sin 2I\frac{1}{2}\cos g\\
&-\sum_{\epsilon=\pm1}\frac{1}{8}\epsilon\sin I(1+\epsilon\cos I)\\
&\times 3\cos(2\lambda-2h-\epsilon g)\Bigg]\\
&+\sin^2 J\Bigg[-\frac{3}{4}\sin^2 I\frac{1}{2}\cos 2g-\frac{1}{16}\\
&\times\sum_{\epsilon=\pm1}(1+\epsilon\cos I)^2 3\cos 2(\lambda-h-\epsilon g)\Bigg]\Bigg)\\
&+\frac{A-B}{4}\Bigg(\sum_{\rho=\pm1}\rho\sin J(1+\rho\cos J)\\
&\times\Bigg[\frac{3}{2}\sin 2I\frac{1}{2}\cos(2\rho l+g)\\
&-\sum_{\epsilon=\pm1}\frac{1}{4}\epsilon\sin I(1+\epsilon\cos I)\\
&\times 3\cos(2\lambda-2h-2\rho\epsilon l-\epsilon g)\Bigg]\\
&+\sum_{\rho=\pm1}\frac{1}{4}(1+\rho\cos J)^2\Bigg[3\sin^2 I\frac{1}{2}\\
&\times\cos(2l+2\rho g)+\sum_{\epsilon=\pm1}\frac{1}{4}(1+\epsilon\cos I)^2\\
&\times 3\cos 2(\lambda-h-\rho\epsilon l-\epsilon g)\Bigg]\Bigg)\Bigg],\end{aligned} \qquad \text{(A.5)}$$

where $\frac{\mathcal{G}M'}{r^3}$ has been replaced by $n^2(\frac{a}{r})^3$.

The variations in the speed of rotation of Venus due to the solid torque exerted by the Sun are obtained by developing Eq. (A.5) as functions of time through the variables M and L_s, taking the eccentricity as a small parameter (Kinoshita 1977). Figure A.1 shows the residuals after substracting of $\frac{\Delta\omega}{\omega}$ obtained by the Eq. (A.5) when the angle J = is taken into account and $e = 0$ with respect to the variations given by Eq. (17) in Sect. 3. We arbitrarily take the angle $J = 0.5°$.

The influence of the angle $J = 0.5°$ on the rotation with amplitudes of $\approx 10^{-9}$ is less than the leading coefficients seen in Eq. (17) by three orders of magnitude and than the influence of the eccentricity of Venus by one order of magnitude. As J is probably much smaller than 0.5° as is the case for the Earth (1″), its influence on the rotation can be neglected with respect to the other effects considered in this paper.

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