NOTES AND CORRESPONDENCE

On Aerosol Direct Shortwave Forcing and the Henyey–Greenstein Phase Function

OLIVIER BOUCHER

Laboratoire d'Optique Atmosphérique, Université de Lille, Lille, France

23 October 1996 and 1 May 1997

ABSTRACT

This technical note extends previous Mie calculations to show that there are complex relationships between the asymmetry parameter g and the upscatter fractions for monodirectional incident radiation $\beta(\mu_0)$. Except for intermediate zenith angles and for the upscatter fraction for diffuse radiation, there are significant differences between $\beta(\mu_0)$ predicted by the Mie theory and that approximated by a Henyey–Greenstein phase function. While the Henyey–Greenstein phase function is widely used in radiative transfer calculations to characterize aerosol or cloud droplet scattering, it may cause important discrepancies in the computation of the aerosol direct radiative forcing, depending on solar zenith angle, aerosol size, and refractive index. The implications of this work for aerosol and climate-related studies are also discussed.

1. Introduction

Atmospheric aerosols have a direct influence on the earth's climate by scattering back to space a fraction of the incident solar radiation (Charlson et al. 1991). One of the current problems in determining the climatic influence of aerosols is an accurate assessment of their direct radiative forcing (Boucher and Anderson 1995). While large uncertainties are due to the lack of reliable data on the spatial and temporal distributions of atmospheric aerosols and their optical properties, there are still uncertainties in the computation of the radiative forcing itself. Aside from the aerosol optical depth and single-scattering albedo (a measure of aerosol absorption), the third important parameter is aerosol phase function. The shape of the phase function varies considerably with particle size and refractive index. As particles grow in size, the phase function evolves from a simple Rayleigh symmetric phase function to a more complex phase function with more and more forward scattering. We discuss in the present note the importance of the phase function to determine aerosol forcing in the light of extensive computations of upscatter fractions.

Corresponding author address: Dr. Olivier Boucher, Laboratoire d'Optique Atmosphérique, UFR de Physique-Bat P5, Université de Lille I, 59655 Villeneuve d'Ascq, France.
E-mail: boucher@loa.univ-lille1.fr

2. The Henyey–Greenstein phase function

The Henyey–Greenstein (HG) phase function was first introduced by Henyey and Greenstein (1941) to describe scattering of radiation in a galaxy. Its expression, with g as single free parameter, is given by

$$P_{\rm HG}(\mu) = \frac{1 - g^2}{(1 + g^2 - 2g\mu)^{3/2}}, \qquad (1)$$

where μ is the cosine of the zenith angle. It has been widely used in atmospheric sciences because of its simplicity. In particular, its Legendre moments

$$\psi_n = \frac{1}{2}\int_0^1 P_{\rm HG}(\mu)P_n(\mu)\,d\mu, \qquad (2)$$

where P_n are the Legendre polynomials, are simply given by

$$\psi_n = g^n, \qquad (3)$$

so that the expansion of the HG phase function in Legendre polynomials is quite simple:

$$P_{\rm HG}(\mu) = \sum_{n=0}^{\infty}(2n + 1)g^n P_n(\mu). \qquad (4)$$

Equation (4) is very convenient in many solutions of the radiative transfer equation like the δ-M method (Wiscombe 1977), the discrete ordinate method (Stamnes et al. 1988), and the Monte Carlo method. It is known that this approximation might cause errors in radiance calculations (Key 1994), but it is generally considered acceptable for flux calculations. Hansen

(1969) indicated that the HG phase function can be used to replace the more realistic Mie phase functions in multiple scattering approximations with no more than a few percent error in computed fluxes. However, Toublanc (1996) showed that using the HG phase function in Monte Carlo radiative transfer simulations of Titan's atmosphere can significantly deteriorate photodissociation rates compared to an improved solution where the Mie phase function is accurately simulated. We further show in the present note that adequate caution should be exercised in using HG phase function to approximate the Mie scattering results.

We compare the realistic Mie phase functions to the corresponding Henyey–Greenstein phase functions with the same asymmetry parameters for two lognormal size distributions with mean geometric volume diameters $D_{gv} = 0.3$ and 3 μm, geometric standard deviation $\sigma_g = 1.4$, and refractive index $m = 1.40 - 0i$. Generally speaking, the HG phase function fails to reproduce the backward scattering peak (glory) and tends to overestimate the Mie phase function at intermediate scattering angles (90°–150°). Coarse mode particles differ from accumulation mode particles in that the HG phase function also overestimates the phase function at scattering angles between a few degrees and 90° (see Fig. 1). The Legendre parameters derived from the Mie phase function differ substantially from Legendre parameters of the HG phase function. For $D_{gv} = 0.3$ μm (see Fig. 2a), Legendre parameters drop much more rapidly for the Mie than for the HG phase function. The opposite is occurring for the more complex phase function of $D_{gv} = 3$ μm (Fig. 2b). For the specific size distributions considered here, the Mie phase function is more readily approximated by a finite sum of Legendre polynomials.

3. Upscatter fractions

a. Definitions

The upscatter fraction for monodirectional radiation incident at zenith angle $\theta_0 = \arccos(\mu_0)$ is defined as

$$\beta(\mu_0) = \frac{1}{2}\int_{-1}^{0}\frac{1}{2\pi}\int_{0}^{2\pi} P(\mu\mu_0 + (1 - \mu^2)^{1/2}(1 - \mu_0^2)^{1/2}\cos\Phi)\, d\Phi\, d\mu. \tag{5}$$

Wiscombe and Grams (1976) reduced this double integral to a single integral over the cosine of the zenith angle μ:

$$\beta(\mu_0) = \frac{1}{2\pi}\int_{\pi/2-\theta_0}^{\pi/2+\theta_0} \arccos(\cot\theta_0 \cot\theta)P(\cos\theta)\sin\theta\, d\theta + \frac{1}{2}\int_{\pi/2+\theta_0}^{\pi} P(\cos\theta)\sin\theta\, d\theta. \tag{6}$$

The backscattered ratio b is no different from the upscatter fraction for radiation incident at zenith angle $\theta_0 = 0$:

$$b = \beta(1) = \frac{1}{2}\int_{-1}^{0} P(\mu)\, d\mu. \tag{7}$$

The upscatter fraction for diffuse radiation (or isotropic upscatter fraction) is further defined as

$$\overline{\beta} = \int_{0}^{1} \beta(\mu)\, d\mu. \tag{8}$$

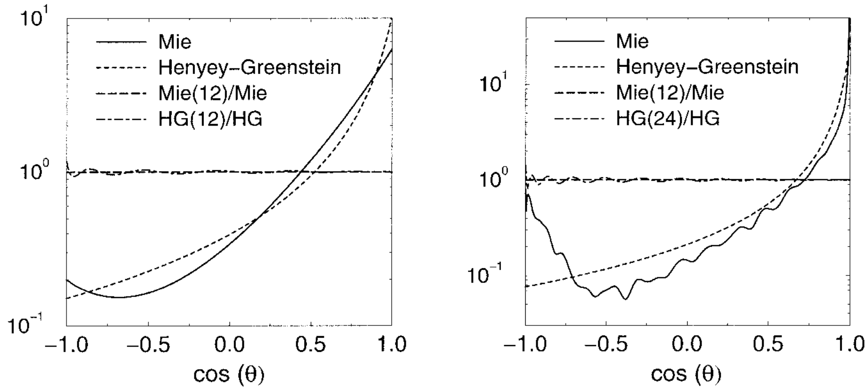
It is also reduced to a single integral by Wiscombe and Grams (1976):

$$\overline{\beta} = \frac{1}{2\mu}\int_{-1}^{1} \arccos(\mu)P(\mu)\, d\mu. \tag{9}$$

b. Previous investigations

The study by Wiscombe and Grams (1976) was motivated by the fact that relationships between $\overline{\beta}$, $\beta(\mu_0)$, and g were necessary to bridge between the Eddington and the two-stream approximations. Moreover, g is a fundamental parameter for radiative transfer in media where multiple scattering dominates (van de Hulst 1974), whereas $\beta(\mu_0)$ is more relevant for a thin aerosol layer where single scattering dominates. The upscatter fraction for diffuse radiation is also the average fraction of the radiation scattered upward by the aerosol when considering the whole illuminated planet. For this reason, it is directly relevant to aerosol forcing of climate and is used for instance by Charlson et al. (1991) and Charlson et al. (1992) to make a first-order estimate of sulfate aerosol forcing. Wiscombe and Grams (1976) showed that $\overline{\beta}$ is an erratic many-valued function of g when individual (spherical) particles were considered. Despite this complexity, $\overline{\beta}_{HG}(g)$ (i.e., the upscatter fraction predicted by the HG function) was found to be a good approximation of $\overline{\beta}$. The error is no more than 2% for individual particles and below 1% for specific size distributions. Wiscombe and Grams (1976) conjectured "that a similar procedure will lead to an approximation for $\beta(\mu)$ of comparable accuracy." This is shown in the next section not to be generally the case.

Marshall et al. (1995) performed extensive Mie calculations and showed the existence of complex relationships between the asymmetry parameter g and the



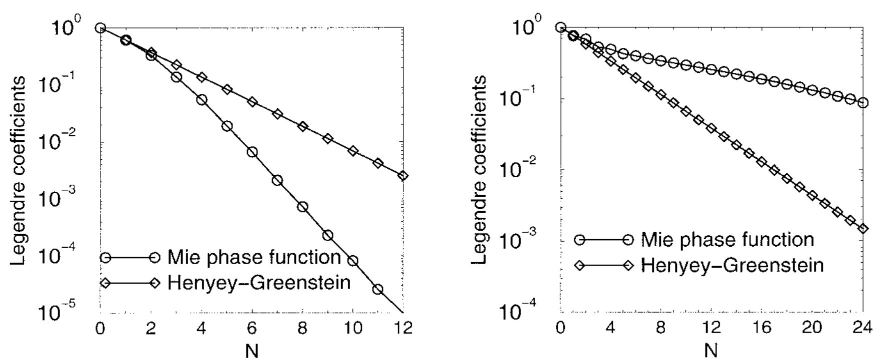
(a) D_{gv}=0.3 μm (b) D_{gv}=3 μm

FIG. 1. Mie phase function (solid line) and the Henyey–Greenstein (HG) phase function with the corresponding asymmetry parameter (dashed line). The curves are for mean geometric volume diameter (a) $D_{gv} = 0.3\ \mu$m and (b) $D_{gv} = 3\ \mu$m, geometric standard deviation $\sigma_g = 1.4$, real refractive index $m = 1.40 - 0i$, and wavelength $\lambda = 0.55\ \mu$m. The respective ratios of the Mie- and HG-approximated phase functions by a sum of 12 (or 24) Legendre moments to the initial phase functions are also depicted on the graphs (Mie: long-dashed line, HG: dotted–dashed line).

backscatter fraction $b = \beta(1)$. This is of direct interest to relate g to nephelometry measurements of b. Again, the relationship between g and b is multivalued, but some empirical one-to-one relationships between the two quantities can be derived for accumulation mode particles provided the breadth of the size distribution is known.

c. Further calculations

Basically, we extended Marshall's calculations to the monodirectional upscatter fractions $\beta(\mu_0)$. We report the relationships between g and $\beta(\mu_0)$ for different μ_0 values and different aerosol size distributions in Figs. 3 and 4. There are substantial discrepancies between Mie-predicted upscatter fractions and upscatter fractions due to the HG phase function with the same asymmetry parameter. Interestingly, the HG phase function can either overestimate or underestimate the upscatter fractions depending on μ_0, refractive index, and aerosol size distribution. We first discuss Fig. 3, where the plots are for a narrow size distribution (σ_g) and a range of real refractive indices. At low μ_0 (i.e., high solar zenith angles, see Figs. 3a,b), the equivalent HG phase function underestimates the value of $\beta(\mu_0)$ for small aerosol diameters but overestimates it for large aerosol diameters. The limit between these two behaviors occurs at about 0.4–0.6 μm for $\lambda = 0.55\ \mu$m, depending on μ_0 and the refractive index. On the contrary, at large μ_0 (i.e., low solar zenith angles, see Figs. 3d,e), the equivalent HG phase function overestimates $\beta(\mu_0)$ for a large range of aerosol diameters (D_{gv} between 0.3 and 1 μm) and underestimates $\beta(\mu_0)$ for aerosol diameters above 1 μm For large diameters, there is not a one-to-one relation-



(a) D_{gv}=0.3 μm (b) D_{gv}=3 μm

FIG. 2. Legendre moments of a Mie phase function ($D_{gv} = 0.3$ or 3 μm, $\sigma_g = 1.4$, $m = 1.40 - 0i$, and $\lambda = 0.55\ \mu$m) and the HG phase function with the corresponding asymmetry parameter.

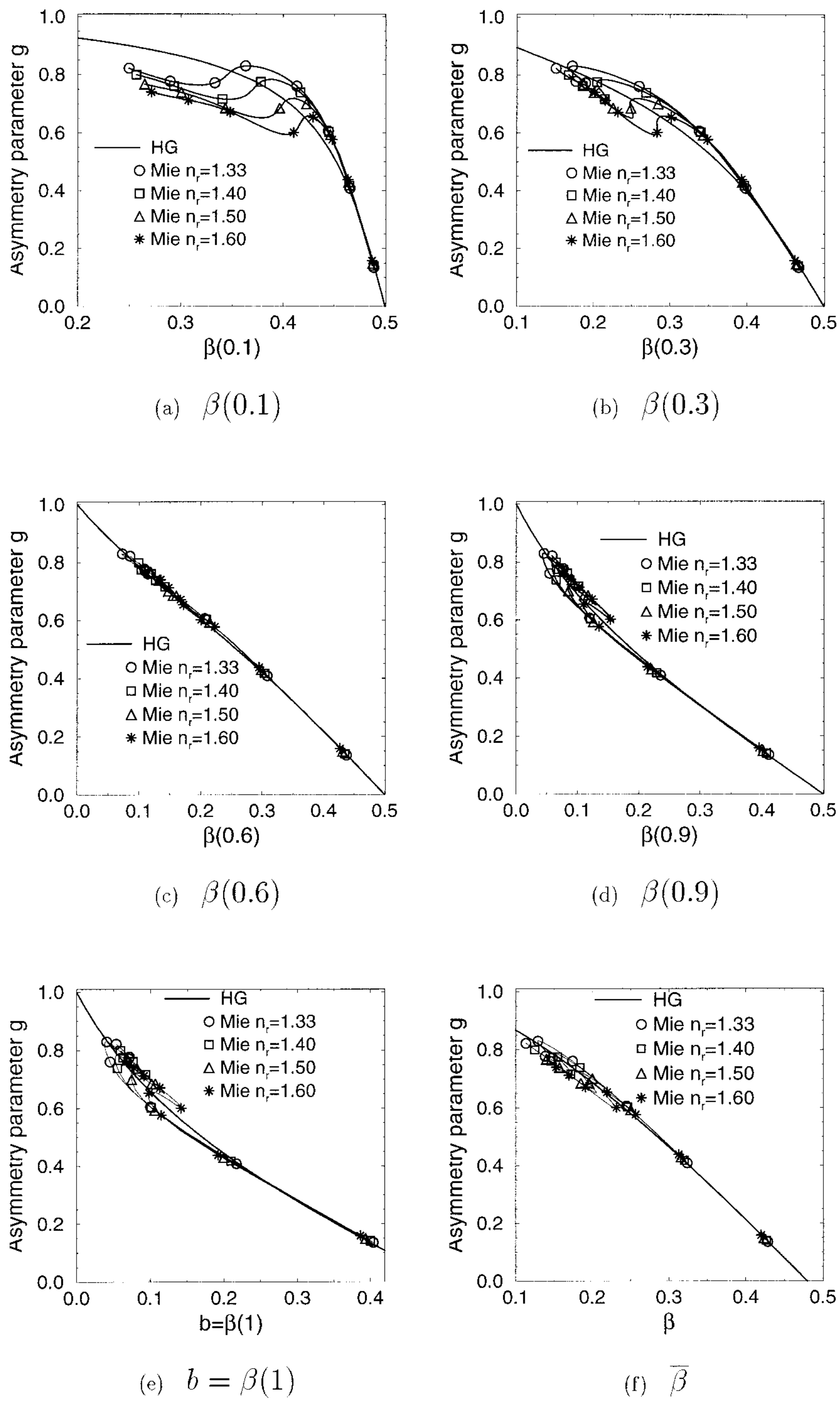


FIG. 3. Asymmetry parameter as a function of (a)–(e) monodirectional upscatter fractions and (f) upscatter fraction for diffuse radiation. We selected lognormal size distributions with mean geometric volume diameters ranging from 0.1 to 5 μm, geometric standard deviation $\sigma_g = 1.4$, and a range of real refractive indices. The wavelength of calculation is $\lambda = 0.55$ μm. The symbols along the curves are for $D_{gv} = 0.1$, 0.2, 0.3, 0.5, 1, 2, 3, and 5 μm from the lower right corner to the upper left corner. The relationships for the HG phase function are shown for comparison (black solid line with no symbol on it). Panel (e) is similar to Fig. 1 of Marshall et al. (1995).

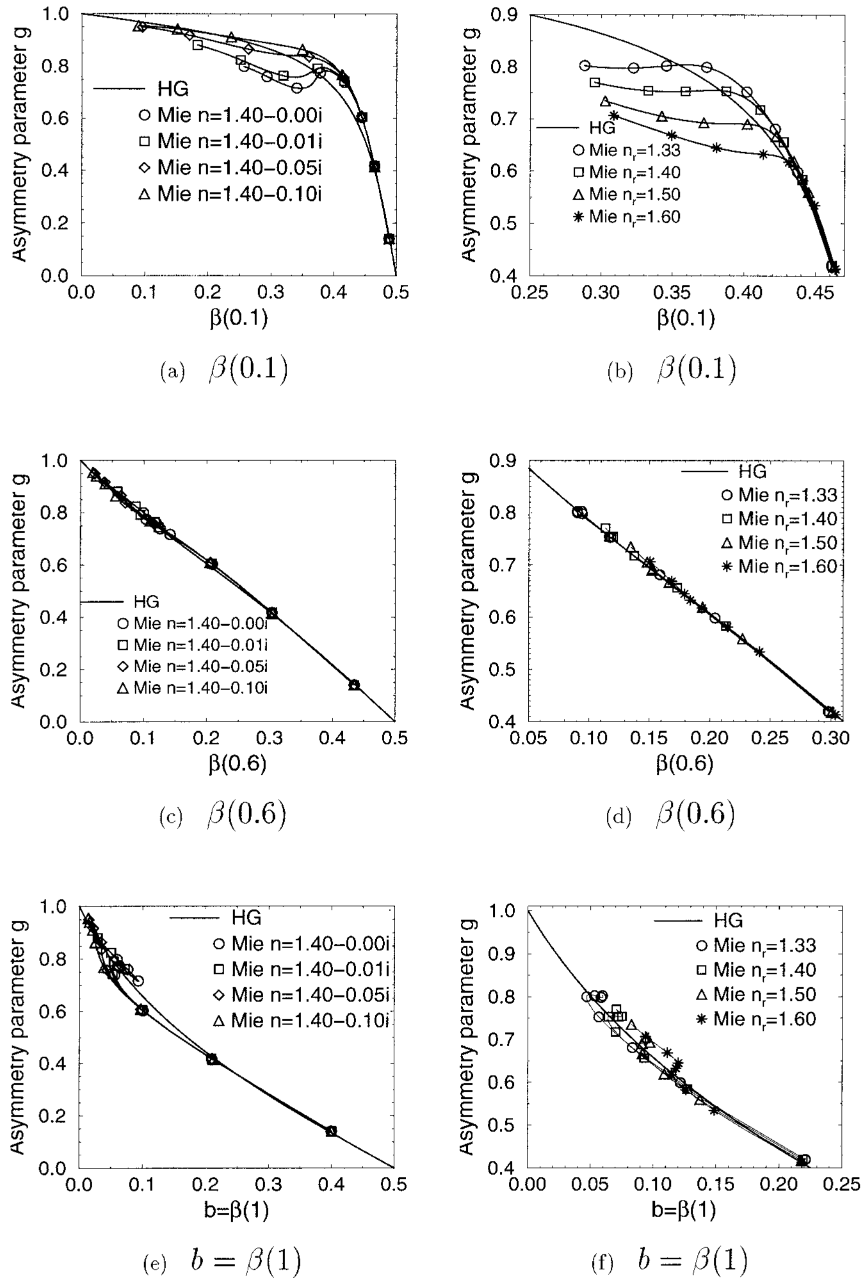
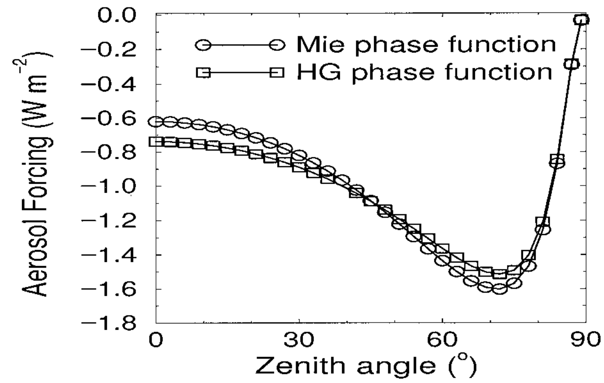


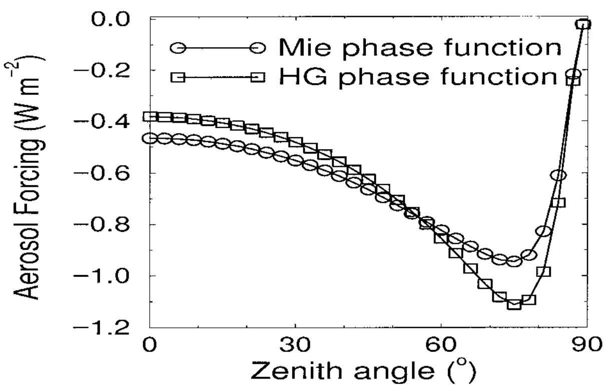
FIG. 4. Same as Fig. 3 but (a), (c), and (e) for geometric standard deviation σ_g = 1.4 and a range of imaginary refractive indices; (b), (d), and (f) for σ_g = 2.0 and a range of real refractive indices. Scales on axes are different from Fig. 3.

ship between g and $\beta(\mu_0)$ and there are up to three upscatter fractions corresponding to a same asymmetry parameter. The agreement between Mie and HG phase functions for the isotropic upscatter fraction $\overline{\beta}$ (Fig. 3f) is generally better because there is a cancellation of the effects at low and high solar zenith angles.

If some absorption is added (Figs. 4a,c,e), no improvement is observed in the prediction of upscatter fractions by the HG phase function for accumulation mode particles. There is, however, a tendency to a better prediction of upscatter fractions for coarse-mode size distributions when a significant amount of absorption is considered. Moreover, the transition between under- and overestimation of the upscatter fractions is progressively translated toward larger aerosol sizes as absorption increases. Broadening the size distribution (Figs. 4b,d,f) flattens the curves a little bit and reduces the disagreement between HG and Mie phase functions for accumulation-mode size distributions but not necessarily for coarse-mode size distributions. Figure 4 thus gives a



(a) $D_{gv} = 0.3\ \mu m$



(b) $D_{gv} = 3\ \mu m$

FIG. 5. Aerosol forcing as a function of solar zenith angle for the Mie and the corresponding HG phase functions for $D_{gv} = 0.3$ and 3 μm, $\sigma_0 = 1.4$, $m = 1.40 - 0i$, and $\lambda = 0.55\ \mu$m. The forcing is computed in the spectral band [0.52, 0.57 μm] for clear-sky conditions, an aerosol optical depth $\tau = 0.1$ and a surface albedo $R_s = 0.1$.

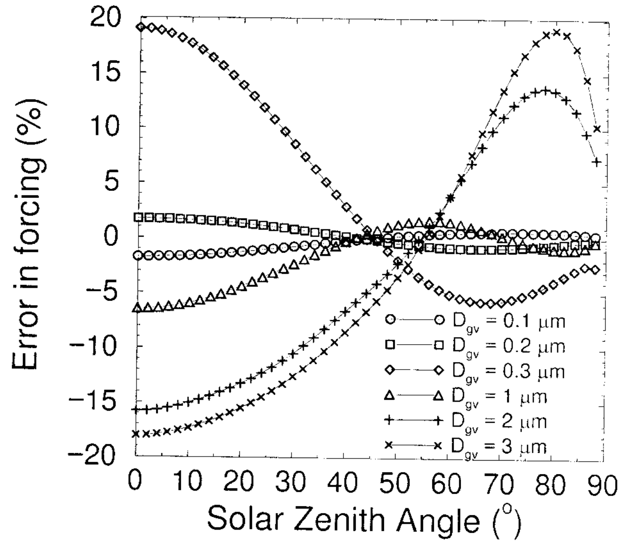


FIG. 6. Relative error of the aerosol forcing computed with the HG phase function compared to the exact forcing computed with the predicted Mie phase function for six particle-size distributions presented in Fig. 3 with real refractive index $m = 1.40 - 0i$. Other parameters are as in Fig. 5.

hint as to how the results presented below (section 4) are affected by the width of the size distribution or the imaginary part of the refractive index.

There is significantly more scatter in these plots than predicted by Cornette and Shanks (1992), who based their Fig. 7 on a limited set of size distributions. In contrast to the conclusion of Cornette and Shanks (1992), this suggests that it is definitively not possible to accurately predict upscatter fractions from a single free parameter, for a wide range of size distributions, whatever the approximated phase function is. The respective behaviors of the Mie-predicted and the HG upscatter fractions are important for computations of the aerosol forcing.

4. Aerosol forcing calculations

The aerosol forcing is computed using the Streamer model (Key 1994 and references therein), itself based on the discrete ordinate method (Stamnes et al. 1988). In these calculations, the HG and the Mie phase functions are approximated by their respective 12 first Legendre moments. The results are not sensitive to the approximation of the aerosol forward-scattering peak (Nakajima and Tanaka 1988), probably because the phase function is well approximated by a series of 12 Legendre moments (Fig. 1). Figure 5 shows that the aerosol forcing goes through a maximum at large solar zenith angles (roughly 70°–80°), in agreement with recent calculations by Nemesure et al. (1995) and Schwartz (1996) for accumulation mode particles. For $D_{gv} = 0.3\ \mu$m, the HG phase function underestimates this maximum while it overestimates the forcing at low solar zenith angles. This is consistent with results from Fig. 3, where the HG phase function underestimates (overestimates) the upscatter fractions at high zenith angles (low zenith angles). The overall effect of the HG phase function, in this specific case, is to diminish the contrast between low and high zenith angles. Figure 6 shows the relative error in the computed aerosol forcings for the same atmospheric conditions as in Fig. 5. The errors are small for small size distributions ($D_{gv} \leq 0.1\ \mu$m) because the HG phase function works well near the Rayleigh scattering regime. For accumulation-mode size distributions ($0.2 < D_{gv} < 0.4\ \mu$m), forcing is significantly underestimated at small zenith angles and significantly overestimated at large zenith angles (up to 20% and consistently with Fig. 6). For coarse-mode particles ($D_{gv} \geq 3\ \mu$m), the errors are equally significant but reversed in sign. For intermediate size distributions ($1 < D_{gv} < 2\ \mu$m), errors are small at the considered wavelengths.

The errors at low and high solar zenith angles do not necessarily cancel out over one diurnal cycle because

i) the distribution of zenith angles experienced at any location depends on latitude and season and ii) other factors that influence aerosol forcing—surface albedo, relative humidity, and cloud cover—also have diurnal variations. In particular, sea surface albedo over a rough sea presents a maximum for solar zenith angles of about 70°–80° (Payne 1972), close to where aerosol forcing is largest. Accurate computations of the diurnal cycle of aerosol forcing are necessary for interpreting the recent decrease in diurnal temperature range over continents (Karl et al. 1991; Karl et al. 1993). A single measure of aerosol phase function (such as *b*) or a parametrization of *g* are clearly not sufficient to make an accurate estimate of aerosol forcing in aerosol column closure experiments (Penner et al. 1994). The breadth of the size distribution and the refractive index for coarse-mode particles are also necessary, as is suggested by Fig. 3. The present study further suggests that careful attention should be placed in the prescription of aerosol phase functions for remote sensing of aerosol properties.

5. Conclusions

While the Henyey–Greenstein (HG) phase function can advantageously replace the Mie phase function in most flux calculations with a small error, it can introduce significant errors (up to 20%) when computing aerosol shortwave radiative forcing (i.e., a difference in fluxes with and without aerosols). The magnitude and sign of the error depend on aerosol size distribution, aerosol refractive index, and solar zenith angle. Furthermore, it is argued that these errors will not necessarily cancel out in the global atmospheric context. Adequate caution should be exercised when using HG phase function as an approximation of Mie-predicted phase functions. If simple radiative transfer models are to be used for aerosol forcing calculations, we suggest that appropriate corrections are made. We further call attention to the need for thoroughly diagnosing the diurnal cycle of aerosol forcing before attributing the recent decrease in surface diurnal temperature range over continents to anthropogenic aerosols.

Acknowledgments. The author would like to thank T. L. Anderson and D. Toublanc for helpful comments and discussions.

REFERENCES

Boucher, O., and T. L. Anderson, 1995: GCM assessment of the sensitivity of direct climate forcing by anthropogenic sulfate aerosols to aerosol size and chemistry. *J. Geophys. Res.,* **100,** 26 117–26 134.

Charlson, R. J., J. Langner, H. Rodhe, C. B. Leovy, and S. G. Warren, 1991: Perturbation of the Northern Hemisphere radiative balance by backscattering from anthropogenic sulfate aerosols. *Tellus,* **43AB,** 152–163.

——, S. E. Schwartz, J. M. Hales, R. D. Cess, J. A. Coakley, J. E. Hansen, and D. J. Hofmann, 1992: Climate forcing by anthropogenic aerosols. *Science,* **255,** 423–430.

Cornette, W. M., and J. G. Shanks, 1992: Physically reasonable analytic expression for the single-scattering phase function. *Appl. Opt.,* **31,** 3152–3160.

Hansen, J. E., 1969: Exact and approximate solutions for multiple scattering by cloudy and hazy planetary atmospheres. *J. Atmos. Sci.,* **26,** 478–487.

Henyey, L. G., and J. L. Greenstein, 1941: Diffuse radiation in the galaxy. *Astrophys. J.,* **93,** 70–83.

Karl, T. R., G. Kukla, V. N. Razuvayev, M. J. Changery, R. G. Quayle, R. R. Heim Jr., D. R. Easterling, and C. Bin Fu, 1991: Global warming: Evidence for asymmetric diurnal temperature change. *Geophys. Res. Lett.,* **18,** 2253–2256.

——, and Coauthors, 1993: A new perspective on recent global warming: Asymmetric trends of daily maximum and minimum temperature. *Bull. Amer. Meteor. Soc,* **74,** 1007–1023.

Key, J., 1994: Streamer user's guide. Cooperative Institute for Research in Environment Sciences Tech. Rep., University of Colorado, Boulder, CO, 71 pp. [Available from Jeff Key, Dept. of Geography, Boston University, Boston, MA 02215, or online from http://stratus.bu.edu.]

Marshall, S. F., D. S. Covert, and R. J. Charlson, 1995: Relationship between asymmetry parameter and hemispheric backscatter ratio: Implications for climate forcing by aerosols. *Appl. Opt.,* **34,** 6306–6311.

Nakajima, T., and M. Tanaka, 1988: Algorithms for radiative intensity calculations in moderately thick atmospheres using a truncation approximation. *J. Quant. Spectrosc. Radiat. Transfer,* **40,** 51–69.

Nemesure, S., R. Wagener, and S. E. Schwartz, 1995: Direct shortwave forcing of climate by anthropogenic sulfate aerosol: Sensitivity to particle size, composition, and relative humidity. *J. Geophys. Res.,* **100,** 26 105–26 116.

Payne, R. E., 1972: Albedo of the sea surface. *J. Atmos. Sci.,* **29,** 959–970.

Penner, J. E., and Coauthors, 1994: Quantifying and minimizing uncertainty of climate forcing by anthropogenic aerosols. *Bull. Amer. Meteor. Soc.,* **75,** 375–400.

Schwartz, S. E., 1996: The whitehouse effect: Shortwave radiative forcing of climate by anthropogenic aerosols. *J. Aerosol Sci.,* **27,** 359–382.

Stamnes, K., S.-C. Tsay, W. Wiscombe, and K. Jayaweera, 1988: Numerically stable algorithm for discrete-ordinate-method radiative transfer in multiple scattering and emitting layered media. *Appl. Opt.,* **27,** 2502–2509.

Toublanc, D., 1996: Henyey–Greenstein and Mie phase functions in Monte-Carlo radiative transfer computations. *Appl. Opt.,* **35,** 3270–3274.

van de Hulst, H. C., 1974: Multiple scattering in cloud layers: Some results. *Proceedings of the UCLA International Conference on Radiation and Remote Probing of the Atmosphere,* J. G. Kuriyan, Ed., Western Periodicals, 162–195.

Wiscombe, W. J., 1977: The delta-*M* method: Rapid yet accurate radiative flux calculations for strongly asymmetric phase function. *J. Atmos. Sci.,* **34,** 1408–1422.

——, and G. W. Grams, 1976: The backscattered fraction in two-stream approximations. *J. Atmos. Sci.,* **33,** 2440–2451.

Notice to Authors: One of the following statements, whichever is appropriate, must be signed by all authors (use multiple forms if needed) of a manuscript and received by the chief editor to which the manuscript is submitted. See "AMS Authors' Guide" (*Bulletin of the AMS*, August 1995). Requests for further information should be addressed to American Meteorological Society, 45 Beacon Street, Boston, MA., 02108. (e-mail: amspubs@ametsoc.org)

AGREEMENT TO TRANSFER COPYRIGHT
(For non-U.S. Government)

Copyright to the article entitled____________________________________

____

by____
assigned and transferred exclusively to the American Meteorological Society (hereinafter referred to as AMS) effective

if and when the article is accepted for publication in__________________________
or in any other AMS journal in which the article may be published with the authors' consent. The authors, however, reserve 1) all proprietary rights other than copyright, such as patent rights, and 2) the right to use all or part of this article in future lectures, press releases, and reviews of their own. Certain additional reproduction rights are granted by sections 107 and 108 of the U.S. Copyright Law. All other uses will be subject to the limitations given in the copyright statement of the AMS journal in which the article is published. To be signed by all authors or if the manuscript is a "work made for hire," by the employer, who is the "legal author" under the U.S. Copyright Law.

Signature | Signature
Print name | Print name
Title if signed by employer | Title if signed by employer
Date | Date

Signature | Signature
Print name | Print name
Title if signed by employer | Title if signed by employer
Date | Date

Note: The form below is to be used for a work of the U.S. Government for which copyright protection is not available under Title 17, Section 105. A work of the U.S. Government is a work prepared by an officer or employee of the U. S. Government as part of this person's official duties.

CERTIFICATION OF U.S. GOVERNMENT MANUSCRIPT

The article entitled____

____

by____
is hereby certified to be a work of the U.S. Government , to have been prepared by the author(s) as officer(s) or employee(s) of the U.S. Government as part of the author(s) official duties, and therefore to be precluded from copyright protection. It is further certified that the article has not been published previously in any language and it is agreed to promptly notify the chief editor if the manuscript is submitted for publication elsewhere before its disposition by the American Meteorological Society.

Signature | Signature
Print name | Print name
Title if signed by employer | Title if signed by employer
Date | Date

Signature | Signature
Print name | Print name
Title if signed by employer | Title if signed by employer
U.S. Government Agency | U.S. Government Agency
Date | Date