Influence of Gravity Waves on the Atmospheric Climate

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1)Dynamical impact of mountains on atmospheric flows

- 2)Representations of mountains in General Circulation Models
- 3)Non-orographic gravity waves sources
- 4)Impact of gravity waves on the middle atmosphere dynamics

1)Dynamical impact of mountains on atmospheric flows

a)Heuristic dimensional analysis

b)Linear mesoscale dynamics

c)Non-interaction theorem

d)Few nonlinear effects

e)Synoptic and planetary scales





Section of the Alps at the latitude ϕ =45.9°N, and according to 3 different dataset The mean corresponds to the truncature of a high resolution GCM (T213, Δx ~75km)



Section of the Alps at the latitude ϕ =45.9°N, zoom to evaluate the scale of the individual mountain peaks: $l \sim 5-10 km$, $h_{max} \sim 1 km$!

In the linear case the response to the mountain can be analysed in terms of Fourier series



The GCM orography resolves the first six harmonics quite well, do the others matter (here the model is T213!).

Linear response in terms of vertical velocity (stationnary 2D response with uniform U and N!), for each harmonics:

 $\frac{d^2 w_K(z)}{dz^2} + k^2 \frac{N^2 - k^2 U^2}{k^2 U^2 - f^2} w_K(z) = 0 \quad \text{with boundary condition:} \qquad w_K(0) = k U h_K$

vertical structure of disturbances with horizontal intrinsic phase velocity : $\hat{c} = -U$ horizontal wavelength $2\pi/k = 2X/K$, and intrinsic frequency: $\hat{\omega} = -U$

$$w(x,z) = -\sum_{K=1}^{K_{f}-1} kUh_{K}e^{-m_{K}z}\sin(kx+X_{K}) - \sum_{K=K_{f}}^{K_{N}}h_{K}\sin(m_{K}z+kx+X_{K}) - \sum_{K=K_{N}+1}^{M}kUh_{K}e^{-m_{K}z}\sin(kx+X_{K})$$

Evanescent « long » disturbances

 $(K_f \approx 4 \pi \frac{f X}{II})$

Gravity waves

Evanescent «short» disturbances

$$(K_N \approx 4 \pi \frac{NX}{U})$$

$$m_k = +k \sqrt{\frac{N^2 - k^2 U^2}{k^2 U^2 - f^2}}$$

where the + sign ensures positive vertical group velocity for the gravity waves and exponential decay with altitude for the evanescent solutions

Heuristic linear analysis, prediction for the mountain drag: $Dr = \frac{1}{2X} \int_{-x}^{x} p \frac{dh}{dx}$

 $Dr = \frac{1}{2} \sum_{K=K_{f}}^{K_{N}} \rho(0) \sqrt{(N^{2} - k^{2} U^{2})(k^{2} U^{2} - f^{2})} h_{K}^{2}$ Only the gravity waves contribute to the drag!

Contribution to the total mountain drag of each harmonics deduced from the IPA dataset (185m x 185m) LAT=45.9 N



$$\sim \sqrt{(N^2 - k^2 U^2)(k^2 U^2 - f^2)}$$

A drag due to the Alps of 1Pa corresponds to a zonal mean tendency of around 1 m/s/day (a barotropic zonal flow of 10m/s is stopped In 10 days). This is large !

The difference between the T213 drag and the High Res datasets drag tells that there is a large need to parameterized Subgrid Scales Orographies!

1) Dynamical impact of mountains on atmospheric flows b) linear mesoscale dynamics (*f*<*kU*<*N*)

The 2D linear analysis of Queney (1947), mountain drag: $Dr = \frac{1}{2X} \int_{-X}^{A} p \frac{dh}{dx}$ $U = 10 m/s, N = 0.01 s^{-1}, f = 10^{-4} s^{-1}$

$$h(x) = \frac{h_{max}}{1 + \frac{x^2}{d}}$$







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Results for the drag as a function of d:



Dynamical impact of mountains on atmospheric flows b) linear mesoscale dynamics (*f*<*kU*<*N*)

Trapped lee-waves and critical levels (U(z) and N(z) varies)

2D-Boussinesq linear non-rotating theory

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} + \left(\frac{N^2}{U^2} - \frac{U_{zz}}{U}\right)w = 0$$
Non
Hydros-
tatic
Fourier decomp: $w(x,z) = \int_{-\infty}^{\infty} \hat{w}(k,z)e^{ikx}dk$

$$\frac{\partial^2 \hat{w}}{\partial z^2} + \left(\frac{N^2}{U^2} - \frac{U_{zz}}{U} - k^2\right)\hat{w} = 0$$
WKB theory: $m_k^2(z) = S(z) - k^2$
Critical level $S(z_c) = \infty, U(z_c) = 0$

WKB theory predicts: $\lim_{z \to z_c} m_k(z) \to \infty$

Breaking and/or dissipation below Z_c



Scorer (1949) + Gossard and Hooke (1975)

Turning heigh:

 $S(z_c) - k^2 = 0$

WKB theory predicts

 $\lim_{z\to z} m_k(z) \to 0$

Total or partial reflection around z_c

Free modes such that w(k,z=0)=0 can be resonantly excited leeding to trapped lee waves

Non-Interaction theorem in the 2D Boussinesq non-rotating framework

$$\theta = \theta_r + \theta_0(z) + \tilde{\theta}(t, x, z), p = p_r(z) + p_0(z) + \tilde{p}(t, x, z)$$

In-flow eqs.:



Momentum budget over the domain [-X,+X] x [h, Z] :

$$\partial_{t} \iint_{-Xh}^{XZ} \rho_{r} u \, dz \, dx + \int_{-X}^{X} \rho_{r} u \, w \, dx + \left[\int_{h}^{Z} (\rho_{r} u \, u + \tilde{p}) \, dz \right]_{-X}^{+X} = - \int_{-X}^{+X} \tilde{p}(h) \frac{dh}{dx} \, dx + \iint_{-Xh}^{XZ} G_{x} \, dz \, dx$$

Dr

Non-Interaction theorem in the 2D Boussinesq non-rotating framework

Momentum budget over a **periodic domain** (again to simplify the maths, but the formalism is general if we take for the temporal derivative the tendency of the large scale flow including advective terms, and include lateral fluxes of momentum):

$$\partial_t \iint_{-XH}^{XZ} \rho_r u \, dz \, dx + \int_{-X}^{X} \rho_r u \, w(Z) \, dx = -Dr + \iint_{-XH}^{XZ} G_x \, dz \, dx$$

In flow momentum budget at a given altitude:

$$\partial_t \rho_r \overline{u} = \partial_z (-\rho_r \overline{u} \overline{w}) + \overline{G_x}$$
 where $\overline{u} = \frac{1}{2X} \int_{-X}^{+X} u \, dx$

What makes $\partial_z (-\rho_r \overline{uw}) \neq 0$?

Linear theory:
$$u = u_0(z) + u'(t, x, z) + \overline{u}(t, z) + \dots$$
; $w = w'(t, x, z) + \dots$
 $O(1)$ $O(\alpha)$ $< O(\alpha^2)$
 α small parameter

Non-Interaction theorem in the 2D Boussinesq non-rotating framework

Disturbance equations written using the vorticity: $\xi' = \partial_z u' - \partial_x w'$



A is the Action (here the pseudo-momentum)

 \vec{F} is the flux of Action

P is the diabatic dissipative production of Action

The budget of action can be written in the conservative form:

$$\partial_t A + \vec{\nabla} \cdot \vec{F} = P$$

Non-Interaction theorem in the 2D Boussinesq non-rotating framework

A being a quadratic quantity in the disturbance amplitude, its sign being also well defined (in the absence of critical levels):

 $\overline{A}(z, t)$ can measures the wave amplitude. Its evolution satisfies:

 $\partial_t \overline{A} + \partial_z \overline{F^z} = \overline{P}$

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For a steady wave, :

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 $\overline{A}(z, t)$ can measures the wave amplitude. Its evolution satisfies:

$\partial_{t}\overline{A} + \partial_{z}\overline{F^{z}} = \overline{F}$

For a steady wave, without dissipative and diabatic effects:

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 $\overline{A}(z, t)$ can measures the wave amplitude. Its evolution satisfies:

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For a steady wave, without dissipative and diabatic effects:

$$\partial_z \overline{F^z} = 0$$

Non-Interaction theorem in the 2D Boussinesq non-rotating framework

A being a quadratic quantity in the disturbance amplitude, its sign being also well defined (in the absence of critical levels):

 $\overline{A}(z, t)$ can measures the wave amplitude. Its evolution satisfies:

$\partial_t \overline{A} + \partial_z \overline{F^z} = \overline{I}$ For a steady wave, without dissipative and diabatic effects: $\partial_{z} \overline{F^{z}} = 0$

At the same level of approximation, the inflow momentum budget at a given altitude becomes:

$$\frac{\partial}{\partial t}\rho_r \overline{u} - \overline{G_x} = \partial_z(\overline{F^z}) = 0$$

The wave does not modify the mean flow in the linear adiabatic non dissipative case (which includes the absence of critical levels and of waves breaking)

From the angular momentum budget $\partial_t \iint_{-X^h}^{X^L} \rho_r u \, dz \, dx + \int_{-X}^{X} \rho_r u \, w \, dx = -Dr + \iint_{-X^h}^{X^L} G_x \, dz \, dx$ and from $\partial_t \rho_r \overline{u} - \overline{G_x} = \partial_z (\overline{F^z}) = 0$, we can deduce that in the same conditions:

 $\frac{1}{2X}\int_{-v}^{u} -\rho_{r}u'w'(Z) dx = \overline{F^{z}} = Dr$ The Eliasen Palm flux at all altitudes balances the surface pressure drag

Consequence of the wave Action conservation law for steady trapped non-dissipated mountain lee waves. $\partial_t A + \vec{\nabla} \cdot \vec{F} = P$

The general wave action law integrated over a non periodic domain when $\partial_t A = P = 0$ gives:

$$\int_{-X}^{+X} F^{z}(Z) \, dx + \int_{0}^{Z} F^{x}(X) \, dz = Dr$$





The trapped lee waves can transport downstream and at low level a substantial fraction of the mountain drag

As the integral $\int_{-x}^{+x} F^{z}(Z) dx$ is little sensitive to X (not shown and as far as X is sufficiently large), we can take for the Eliasen Palm flux vertical profile the one predicted by a linear non-dissipative theory, and assumes that the horizontal transport at low level is dissipated.



Linearity condition :
$$Z_B >> h_{max}$$

From the vertical wavenumber definition:

$$m_k = +k \sqrt{\frac{N^2 - k^2 U^2}{k^2 U^2 - f^2}}$$

$$Z_B$$
 can be written:
 $Z_B \sim \frac{\pi}{2m_{1/d}} = \frac{\pi d}{2} \sqrt{\frac{1 - Ro^{-2}}{Fr^{-2} - 1}}$

For large Rossby number the linearity condition writtes:

$$h_{max}/d\sqrt{Fr^{-2}-1} \ll 1$$

Neutral or Fast Flows :

The linearity condition becomes (for large Rossby number)

$$Fr^{-1} = \frac{Nd}{U} \ll 1$$

$$h_{max}/d\sqrt{|Fr^{-2}-1|} \sim \frac{h_{max}}{d} = S \ll 1$$

S is the slope parameter, it is almost never small!



Fig 4 Slope of the mountain averaged over 10'x10' boxes covering all the alpine regions. Y-axis, values deduced from the USN dataset. X-axis: values deduced from the high resolution (IPA) dataset.

Neutral or Fast Flows :

$$Fr^{-1} = \frac{Nd}{U} \ll 1$$

Nonlinear dynamics for
$$S=h_{max}/d\sim O(1)$$

S is the slope parameter, it is almost never small!



Figure 1. The model-derived streamlines for flow over the two-dimensional hill with h = 200 m ($\lambda = 1000$ m and $Z_0 = 0.1$ m). The vertical axis is linear in height above the upstream surface. The horizontal axis shows distance from the point on the upstream slope at which the hill height is half of its maximum value. A separation streamline is clearly visible.

(if the valley is ventilated!)

Neutral or Fast Flows :



Figure 1: Banner clouds forming leeward of a pyramidal shaped mountain peak or a quasi 2D ridge. (a) Banner cloud at Matterhorn (Switzerland). (b) Banner cloud at Mount Zugspitze (Bavarian Alps). Mean flow from right to left.

$$Fr^{-1} = \frac{Nd}{U} \ll 1$$

Nonlinear dynamics for $S = h_{max}/d \sim O(1)$

The dynamics at these scales explain the formation of the « banner » clouds alee of elevated and narrow mountain ridges (Reinert and Wirth, BLM 2009)



Stratified or « slow » Flows $Fr^{-1} = \frac{N d}{II} \gg 1$

The linearity condition becomes (for large Rossby number)

$$h_{max}/d\sqrt{|Fr^{-2}-1|} \sim \frac{h_{max}N}{U} = H_{ND} \ll 1$$

 H_{ND} is the non-dimensional mountain height, again it is almost never small!



Stratified or « slow » Flows $Fr^{-1} = \frac{N d}{U} \gg 1$ Single obstacle simulation with $h \sim 1$ km, U = 10 m/s, N=0.01s⁻¹ : $H_{ND} = 1$! (Miranda and James 1992) Note:



Quasi vertical isentropes at low level downstream: Wave breaking occurs.

The strong Foehn at the surface downstream

Residual GWs propagating aloft

Apparent slow down near the surface downstream, And over a long distance

Question:

How to quantify the reversible motion that is due to the wave, from the irreversible one that is related to the breaking and that affect the large scale?

Answer:

On the Potential Vorticity (I am sure Ron Smith did a Very good job in explaining that to you)

Analogy between low level breaking waves and Hydraulic jumps in shallow water flow (Schar and Smith 1992)



The Potential vorticity tells where the waves is dissipated and/or break: a steady non-dissipative wave do not produce Potential Vorticity anomalies

Note the analogy with the non-interaction theorem

The effect of mountains and obstacles is to produce an inflow force (F,G) at the place where the surface pressure drag due to the presence of the waves is given back to the flow. That is where the waves break.

A very comparable effect occurs when The mountain intersect the free surface in shallow water, or intersect isentropes In the continuously startified case

Analogy between low level breaking waves and Hydraulic jumps in shallow water flow (Schar and Smith 1992), case where the mountain pierces the free surface.



Basic mechanism:



The vortex compression over large scale mountains produces anticyclonic circulations (linear, QG, non-dissipative view):

 $\frac{d\left(\xi\!+\!f\right)\!\theta_z}{dt}\!=\!0$

QG vorticity:

$$\boldsymbol{\xi} = \partial_x v_g - \partial_y u_g$$

No diabatic effect and the surface stays an isentrope (inherent to a steady linear theory):

$$\frac{d\,\theta}{dt} = 0$$

Basic mechanism:



The force exerted on the mountain is almost perpendicular to the low level incident wind (like a lift force!).

It follows that the mountain felt the backgound pressure meridional gradient in geostrophic equilibrium with the background wind :

$$\vec{D}r = \frac{1}{4XY} \int_{-Y}^{Y} \int_{-X}^{X} + p \vec{\nabla} h \, dx \, dy$$

In the linear case:

$$\vec{D}r = \rho f U \bar{h} \vec{y}$$

A case of lee-cyclogenesis (NCEP Data)



Geop. Height (dm) at 850mb year 2003, day 23 Geop. Height (dm) at 850mb year 2003, day 25 Geop. Height (dm) at 850mb year 2003, day 27

This basic mechanism is in part operand during the triggering of lee Cyclogenesis. The lift force therefore needs to be well represented in models

Composites of surface maps keyed to -D_v (NCEP Data, 40 cases out of 28 years)



Almost the structure of the cold surges.

Patterns affecting cold surge onset and monsoon precipitations (not shown, but very significant over the South-China sea)

SFP anomaly (contours, Student index), significant surfacetem perature anomalies (blue=cold, red=warm) Mailler and Lott (2009, 2010)

The peaks of -D_y are due high pressures and low temperatures over the Tibet to the North of the Himalayas

These structures are few days systematic precusors of the *Cold Surges*, the cold surges ptoducing high negative values in D_x

How can a lateral force supplement the orographic forcing?



U_g: Geostrophic wind Dr: Drag Mountain is in green P_s: Surface pressure

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G=-Dr: the force on the atmosphere is opposite to the drag U_a ageostrophic wind: Equilibrate **G** via Coriolis Transport mass from left to the right here

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Winter planetary stationnary wave



NCEP reanalysis, géopotentiel à 700hPa, average over winter months