The partitionning and coupling methodology

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Model coupling

Problem studied in many science areas, with various purposes.

- black box coupling ; that is coupling between very complex models, where nobody is expected to know all components.
- building complex models from elementary items.
- matching sub-models issued from model splitting (e.g. boundary layer).

Tools relevant for such opaque systems: sensitivity analysis, coupling analysis, feedback analysis.

Scientific issues

- dealing with non-linearities, shocks, extreme events ...
- feedback analysis ($\equiv$ analysis of Tangent Linear System)

Models

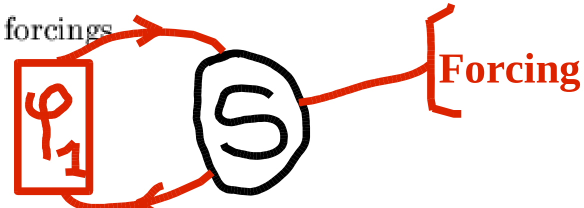
appearance
physics, chemistry, biology, economy
mathematics
numerics
software

Mathematics = common language (hopefully)

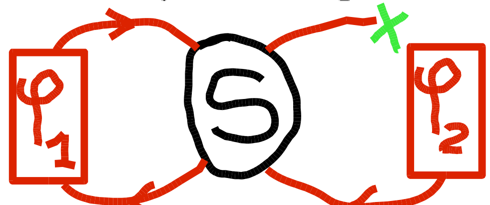
- we consider models belonging to mathematics and numerics worlds:
  Models = sets of equations
- always try to exhibit links with upper layers
- (Sub-)System = model $\oplus$ variables

Sensitivity, coupling, feedback

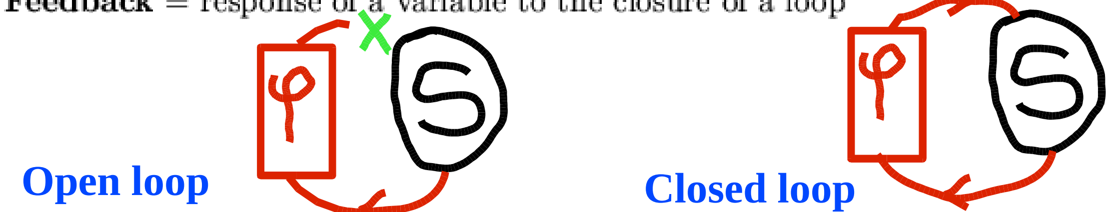
Sensitivity = response to a change in forcings



Coupling = response of a variable φ_1 to a change of model relative to another variable φ_2

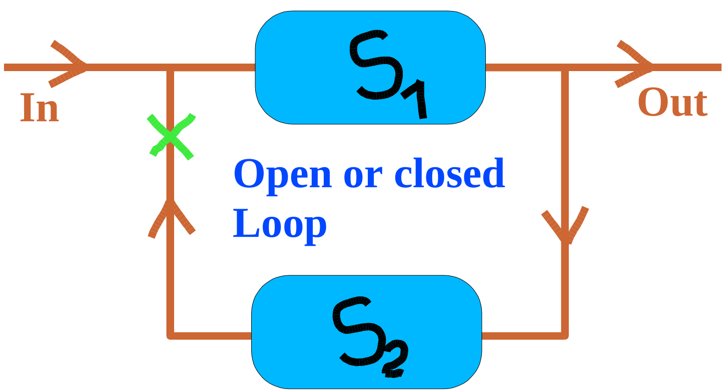


Feedback = response of a variable to the closure of a loop



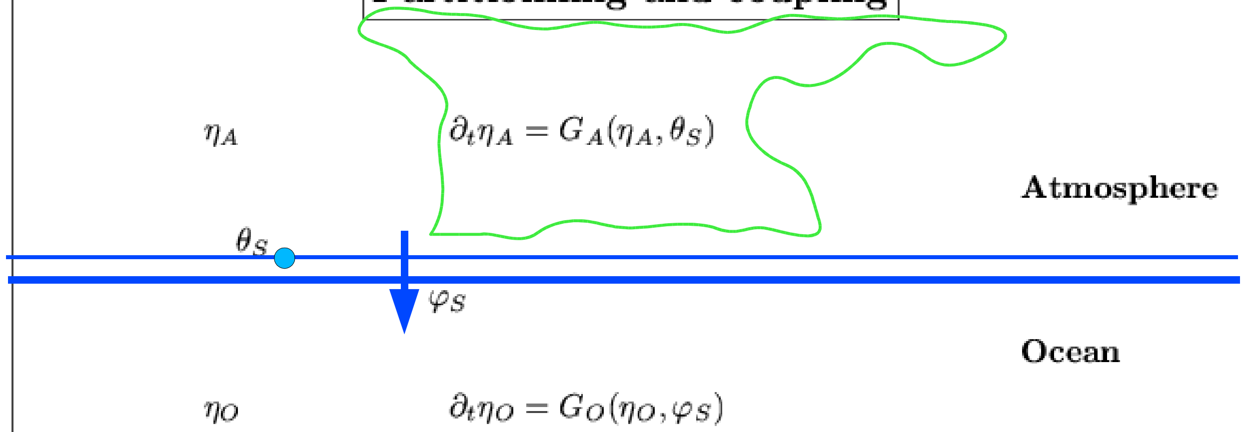
Importance of partitionning into labelled submodels

All this looks quite different from the usual scheme :



Alain Lahellec will show that all these schemes are closely akin.

Partitionning and coupling



- For every sea surface temperature $\theta_S \longrightarrow$ one (unique) solution for η_A
- For every surface flux $\varphi_S \longrightarrow$ one (unique) solution for η_O

Transfer models = Supplementary conditions restoring the behaviour of the complete system.

Usual coupling technique in GCMs :

- temperature continuity: $\theta_S = T_{S(\eta_O)}$
- flux continuity at surface: $\varphi_S = \Psi_{S(\eta A)}$

TEF ; global structure

Two kinds of models

- cells = models relative to parts of the system (partial models ; well posed problems)
- transfers = coupling and matching models

Two sets of variables

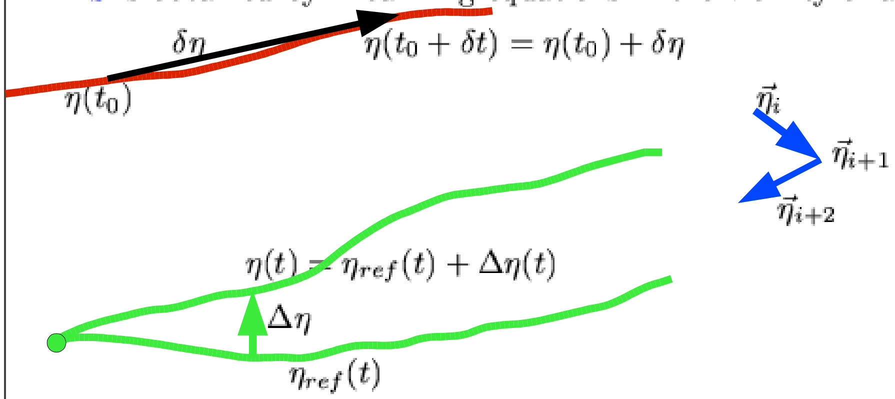
- state variables ($\vec{\eta}_\alpha$)
- transfer variables ($\vec{\varphi}$)

Equations :

Unsteady model		steady model
$\partial_t \vec{\eta}_\alpha = \vec{G}_\alpha(\vec{\eta}_\alpha, \vec{\varphi})$	for each cell α	$\vec{G}_\alpha(\vec{\eta}_\alpha, \vec{\varphi}) = 0$
$\vec{\varphi} = \vec{f}(\{\vec{\eta}_\alpha\})$	for transfers	$\vec{\varphi} = \vec{f}(\{\vec{\eta}_\alpha\})$

TLS: Tangent Linear System

TEF is mainly designed to study properties of the Tangent Linear System (TLS).
TLS is obtained by linearizing equations in the vicinity of a reference state $\vec{\eta}_{ref}$.



TLS is used in three ways:

- for dynamic models and small time displacement: TLS is used in the time marching simulation procedure.
- for steady model iterative solving procedure: Newton Raphson method makes use TLS.
- for pertubation studies.

Transfers

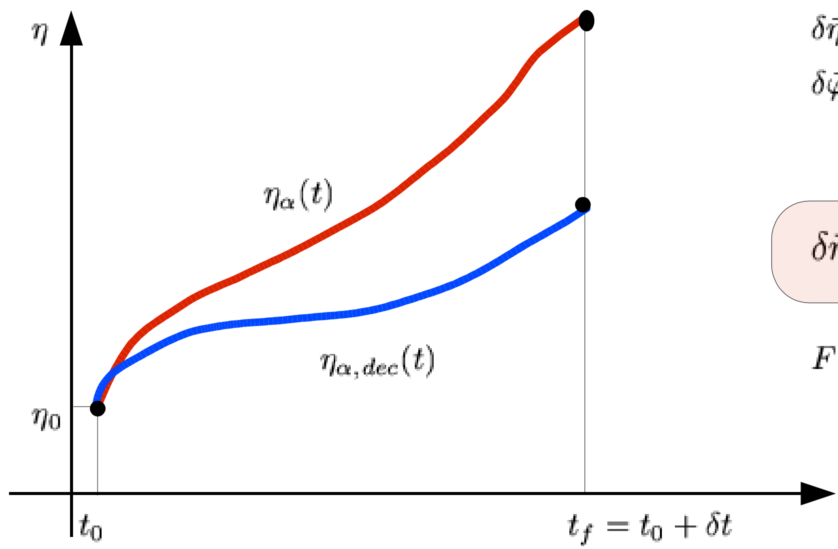
$$\vec{\varphi} = \vec{f}(\{\vec{\eta}_\alpha\})$$

$$\delta\vec{\varphi} = \sum_\alpha \partial_\alpha f \ . \ \delta\vec{\eta}_\alpha$$

where: $\partial_\alpha f = \dfrac{\partial \vec{f}}{\partial \vec{\eta}_\alpha}$

Cells

Unsteady model		steady model
$\partial_t \vec{\eta}_\alpha = \vec{G}_\alpha(\vec{\eta}_\alpha, \vec{\varphi})$	for each cell α	$\vec{G}_\alpha(\vec{\eta}_\alpha, \vec{\varphi}) = 0$



$$\delta\vec{\eta} = \vec{\eta}(t_0 + \delta t) - \vec{\eta}(t_0)$$
$$\delta\vec{\varphi} = \vec{\varphi}(t_0 + \delta t) - \vec{\varphi}(t_0)$$

$$\delta\vec{\eta}_\alpha = \delta\vec{\eta}_{\alpha,dec} + F \ . \ \delta\vec{\varphi}$$

F = influence matrix

Coupling equation: state variable elimination

State variable elimination:

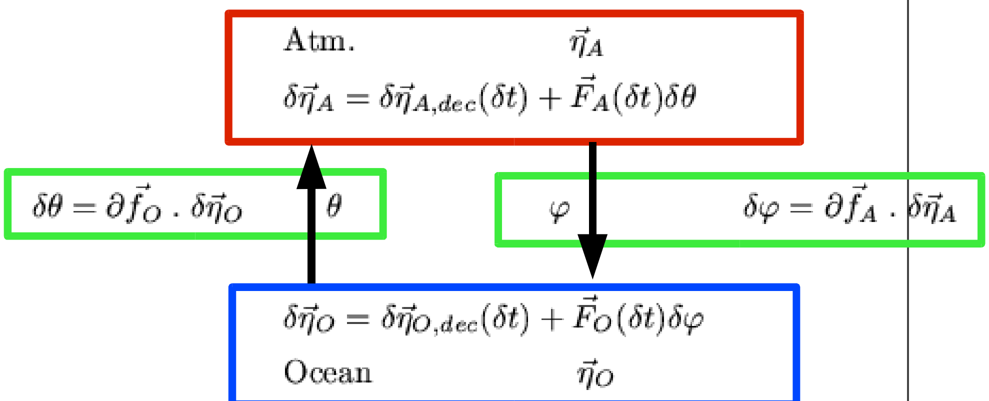
$$\begin{cases} (1 - \sum_\alpha \partial_\alpha f \ . \ F_\alpha)\delta\vec{\varphi} = \delta\vec{\varphi}_{ins} \\ \delta\vec{\varphi}_{ins} = \sum_\alpha \partial_\alpha f \ . \ \delta\vec{\eta}_{\alpha,dec} \end{cases}$$

Time discretized feedback gains

Example : **Ocean/atmosphere thermodynamic coupling**

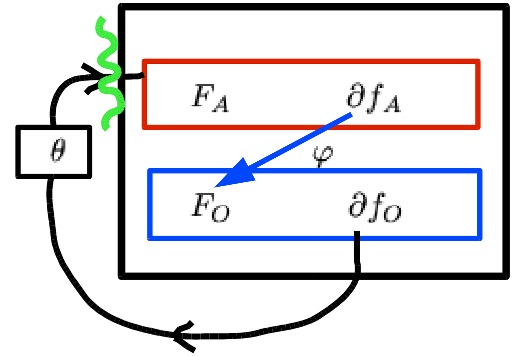
1D problem with 2 interface variables (sea surface temperature and heat flux).

Question: is the feedback important ? what constraint does it impose on the coupling method ?



Solving for $\delta\theta$ yields:

$$\begin{cases} (1-g)\delta\theta = \delta\theta^*_{ins} \\ g = \partial\vec{f}_O \,.\, \vec{F}_O \; \partial\vec{f}_A \,.\, \vec{F}_A \\ \delta\theta^*_{ins} = \partial\vec{f}_O \,.\, \delta\vec{\eta}_{O,dec} + \partial\vec{f}_O \,.\, \vec{F}_O \; \partial\vec{f}_A \,.\, \delta\vec{\eta}_{A,dec} \end{cases} \tag{2}$$



Open loop: system made insensitive to θ by setting $\vec{F}_A = 0$. Equation reads $\delta\theta = \delta\theta^*_{ins}$.

Closed loop: the gain g comes in: g describes the effect of the feedback.

The gain is negative (if the flux increases, the ocean warms up; if the surface temperature increases, the flux decreases); it is the ratio of the atmospheric conductance to the ocean conductance.

For a time step of 30', if the ocean is free of ice, the gain is of the order of -0.1.
Sea-ice isolates the ocean and may cause the gain to reach the [-10,-100] range.
When |g| >1, simple relaxation diverges; somethingelse has to be done . . .

Dynamic feedback analysis

Exact solving of TLS = Use of Laplace or Borel Transform

Borel transform:

$$\mathcal{B}(f)_{(\tau)} = \tilde{f}_\tau = \frac{1}{\tau}\int_0^\infty exp(-\frac{t}{\tau}) f(t) dt \tag{3}$$

Why Borel transform ? Because it is very easy to compute: any computer code, that uses Crank-Nicolson scheme to compute $\delta\varphi^{CN}(\delta t)$ (such as ZOOM), can compute numerically $\delta\tilde{\varphi}(\tau)$,thanks to:

$$\tilde{f_{(\tau)}} = \frac{1}{2} f^{CN}_{(2\tau)}$$

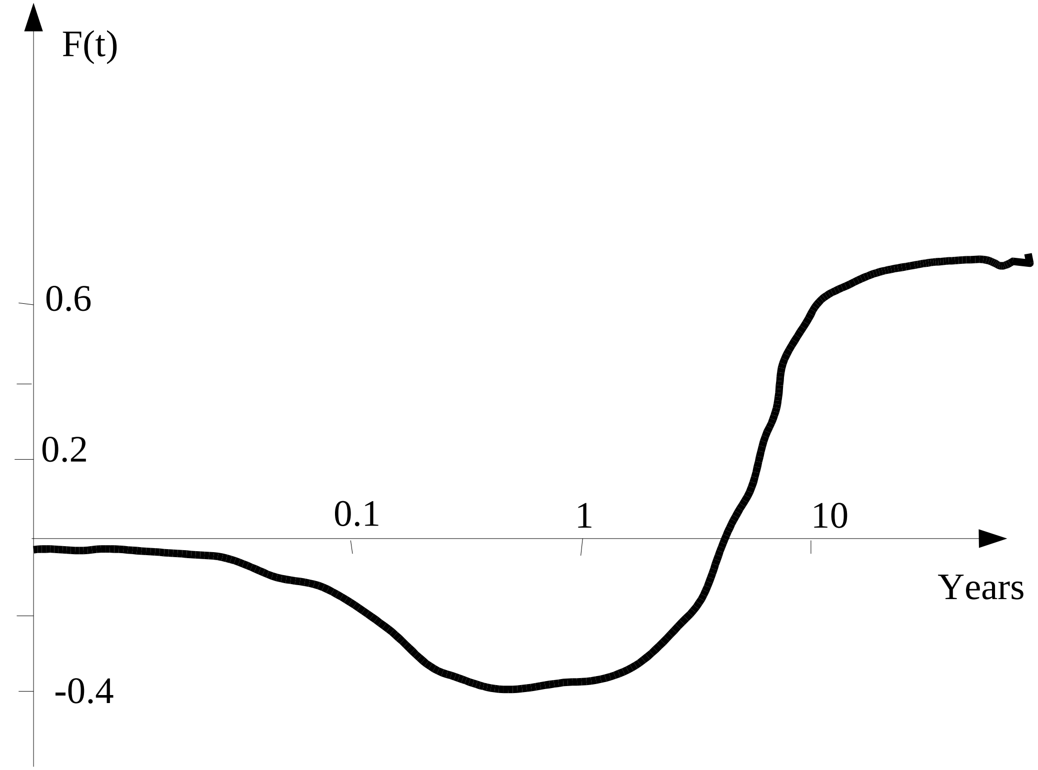
The discrete time coupling equation translates directly in a Borel time one:

$$(1-g^{CN})\delta\vec{\varphi}^{CN} = \delta\vec{\varphi}^{CN}_{ins} \quad \longrightarrow \quad (1-\tilde{g}(\tau))\delta\tilde{\vec{\varphi}}(\tau) = \delta\tilde{\vec{\varphi}}_{ins}(\tau)$$

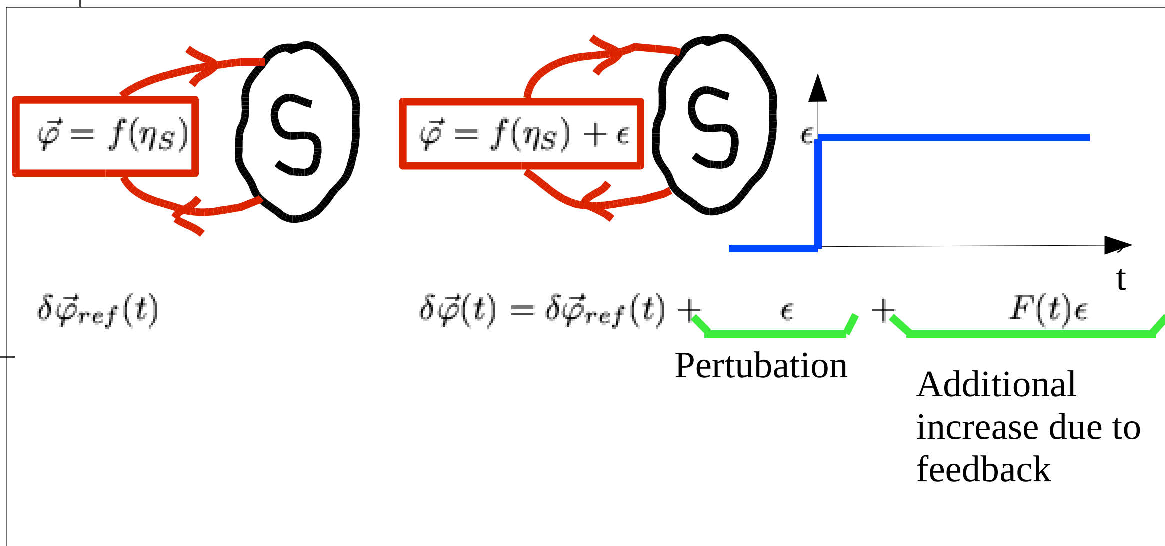
Back in real space, the evolution of $\vec{\varphi}$ reads:

$$\delta\vec{\varphi}(t) = \delta\vec{\varphi}_{ins}(t) + F(t) * \frac{\partial(\delta\vec{\varphi}_{ins}(t))}{\partial t}$$

with : $F(t) = \mathcal{B}^{-1}(\frac{\tilde{g}}{1-\tilde{g}})_{(t)}$



It takes time for feedbacks to settle to their final value.



Summing up

- model splitting and coupling is an efficient modelling tool
- model splitting enables coupling and feedback analysis
- importance of studying the Tangent Linear System
- feedback gains are properties of the TLS (not effects of non-linearities)
- feedback gains describe the effect of the closure of a feedback loop
- full dynamic analysis of TLS yields characteristic times as well as values of feedback gains

Issues

- application to economy-climate coupling ($->$ Stephane Hallegatte)
- application to Global Climate models
- dealing with non linearities, shocks ...