Radiative Net Exchange Formulation Within One Dimensional Gas Enclosures With Reflective Surfaces

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1. Introduction

The Net Exchange Formulation (NEF) and the Exchange Monte Carlo Method (EMCM) were proposed in (Cherkaoui et al., 1996) for accurate computation of infra red radiative exchanges within gas enclosures. This formulation has some common principles with the exchange factor methods (Zone Method...) although the volume are not supposed to be isothermal and the fundamental variable is not the exchange but the net-exchange flux.

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For grey absorbing-emitting/isotropically-scattering medium confined in a Lambert enclosure, the totale exchange areas are completly defined by (Noble, 1975). To deal with non-grey medium surrounded by reflective surfaces, the radiative exchanges should be followed along a lot of reflections or scatterings. Up to now, only low order closure algorithm were used (Naraghi and Huan, 1991; Menart et al., 1993). The aim of this technical note is to adresse, in a simple problem (no scattering, one dimensional) but for a wide range of optical properties, the exact solution of the radiative exchanges.

The extension of the NEF to the case of reflective surfaces (Sec. 2) and the derivation of an estimator of the truncation error (Sec. 3) are first presented. Optically thin gas problems with highly reflective surfaces are then studied (Sec. 4).

2. Generalization of the Net-Exchange Formulation with multiple reflections.

In the Net Exchange Formulation, radiative transfer is described in terms of Net Exchange Rates (NER) $\Psi(i,j)$ between zone pairs i,j. The radiation budget $\Psi(i)$ of one zone i is the sum of all NER between i and the other zones of the discretized system.

$$\Psi(i) = \sum_j \Psi(i,j) \tag{1}$$

The configuration considered in this note is a one-dimensional slab with reflective surfaces. Under the assumption that the gas refraction index is uniform, any optical path γ is a set of broken lines and is entirely defined with the abscissae x_γ and y_γ of origin and end, the number r_γ of surface reflections, the vector $\vec{\mu}_\gamma$ of the segment cone angle cosines (the dimension of $\vec{\mu}_\gamma$ is $r_\gamma + 1$) and, in the case of the exchange between two gas volumes, the first encountered surface number n_γ (see Fig. 1) :

$$\gamma = \Gamma(x_\gamma, y_\gamma, r_\gamma, \vec{\mu}_\gamma, n_\gamma) \tag{2}$$

The monochromatic optical depth of the path γ is given as

$$\tau_{\nu,\gamma} = \begin{cases} \tau_\nu\left(x_\gamma, y_\gamma, \mu_\gamma(0)\right) & \text{if } r_\gamma = 0, \\ \tau_\nu\left(x_\gamma, S(n_\gamma), \mu_\gamma(0)\right) + \sum_{s=1}^{r_\gamma - 1} \tau_\nu\left(S(1), S(2), \mu_\gamma(s)\right) + \tau_\nu\left(S(M_\gamma(r_\gamma)), y_\gamma, \mu_\gamma(r_\gamma)\right) & \text{if } r_\gamma \geq 1 \end{cases} \quad (3)$$

where $\tau_\nu\left(x, y, \mu\right)$ is the optical depth of the x to y segment with a cone angle cosine μ, $S(1)$ and $S(2)$ are the abscissae of the two surfaces and $M_\gamma(s)$ is the index of the surface encountered at the s-th reflection.

The monochromatic directional-directional reflectivity of the m-th surface is noted

$$\rho_\nu^{dd}(\mu_r, \mu_i, m) = \rho_\nu^{dh}\left(\mu_i, m\right) q_\nu\left(\mu_r; \mu_i, m\right) \quad (4)$$

where ρ^{dh} is the directional-hemispheric reflectivity and q is the probability density function of the reflection cosine μ_r for an incident cosine μ_i. The compound reflection factor $R_{\nu,\gamma}$ is then defined as the directional-directional reflectivity product for all reflections along the path γ :

$R_{\nu,\gamma} = 1$ if $r_\gamma = 0$

$R_{\nu,\gamma} = \prod_{s=1}^{r_\gamma} \rho_\nu^{dh}\left(\mu_\gamma(s-1), M_\gamma(s)\right)\ q_\nu\left(\mu_\gamma(s); \mu_\gamma(s-1), M_\gamma(s)\right)$ if $r_\gamma \geq 1$

We now consider a gas layer i between abscissae X_i and X_{i+1} and a gas layer j between abscissae X_j and X_{j+1}. The monochromatic NER between i and j (see Cherkaoui et al., 1996) can be expressed by summing the contributions of optical paths involving r reflections with r varying from zero to infinity :

$$\psi_\nu^{gg}(i,j) = \sum_{r=0}^{\infty} \int_{X_i}^{X_{i+1}} dx \int_{X_j}^{X_{j+1}} dy \int_0^1 d\mu(0) ... \int_0^1 d\mu(r) \ \left\{ \sum_{n=1}^{2} h_\nu^{gg}\left(\Gamma(x, y, r, \vec{\mu}, n)\right) \right\} \quad (5)$$

where

$$h_\nu^{gg}\left(\gamma\right) = \pi\ \left[B_\nu(y_\gamma) - B_\nu(x_\gamma)\right]\ 2\ \mu_\gamma(0)\ R_{\nu,\gamma} \left| \frac{\partial^2 exp(-\tau_{\nu,\gamma})}{\partial x_\gamma \partial y_\gamma} \right| \quad (6)$$

Two assumptions are made when averaging monochromatic NER over a spectral narrow band : monochromatic black body intensities and reflection properties are assumed uniform ($B_\nu \approx \overline{B}$,

$\rho_\nu^{dh} \approx \overline{\rho}^{dh}$ and $q_\nu \approx \overline{q}$). With only these two assumptions and by inverting angular and spectral integrals, the average over a narrow band l of the NER between two gas layers is :

Gas-gas exchanges

$$\overline{\psi}_l^{gg}(i,j) = \sum_{r=0}^{\infty} \int_{X_i}^{X_{i+1}} dx \int_{X_j}^{X_{j+1}} dy \int_0^1 d\mu(0) ... \int_0^1 d\mu(r) \left\{ \sum_{n=1}^{2} \overline{h}^{gg}\Big(l, \Gamma(x,y,r,\vec{\mu},n)\Big) \right\} \tag{7}$$

where

$$\overline{h}^{gg}\Big(l,\gamma\Big) = \pi \ \left[\overline{B_l}(y_\gamma) - \overline{B_l}(x_\gamma)\right] \ 2 \ \mu_\gamma(0) \ \overline{R}_{l,\gamma} \ \left| \frac{\partial^2 \overline{F}_l(\gamma)}{\partial x_\gamma \partial y_\gamma} \right| \tag{8}$$

$$\overline{F}_l(\gamma) = \frac{1}{\Delta\nu} \int_{\Delta\nu} exp(-\tau_{\nu,\gamma}) d\nu \tag{9}$$

$\overline{F}_l$ is defined as the spectral average transmission function, which can be approximated using narrow band models. The NER between surface m and gas layer i and between surface m and surface m' can be formulated similarly :

Surface-gas exchanges

$$\overline{\psi}_l^{sg}(m,i) = \sum_{r=0}^{\infty} \int_{X_i}^{X_{i+1}} dy \int_0^1 d\mu(0) ... \int_0^1 d\mu(r) \ \overline{h}^{sg}\Big(l, \Gamma(S(m),y,r,\vec{\mu},m')\Big) \tag{10}$$

where m' is the index of the surface facing m and

$$\overline{h}^{sg}\Big(l,\gamma\Big) = \pi \ [1 - \overline{\rho}_l^{dh}(\mu_\gamma(0),m)] \ \Big[\overline{B_l}(y_\gamma) - \overline{B_l}(S(m))\Big] \ 2 \ \mu_\gamma(0) \ \overline{R}_{l,\gamma} \ \left| \frac{\partial \overline{F}_l(\gamma)}{\partial y_\gamma} \right| \tag{11}$$

Surface-surface exchanges

$$\overline{\psi}_l^{ss}(m,m') = \sum_{r=0}^{\infty} \int_0^1 d\mu(0) ... \int_0^1 d\mu(r) \ \overline{h}^{ss}\Big(l, \Gamma(S(m),S(m'),r,\vec{\mu},m')\Big) \tag{12}$$

where

$$\overline{h}^{ss}\Big(l,\gamma\Big) = \pi \ [1 - \overline{\rho}_l^{dh}(\mu_\gamma(0),m)][1 - \overline{\rho}_l^{dh}(\mu_\gamma(r_\gamma),m')] \times \Big[\overline{B_l}(S(m')) - \overline{B_l}(S(m))\Big] \ 2 \ \mu_\gamma(0) \ \overline{R}_{l,\gamma} \ \overline{F}_l(\gamma) \tag{13}$$

3. Truncation error over-estimates

Preceding NER expressions involve infinite sums over surface reflections. The present section aims at finding an estimate of the errors made when omitting higher order terms. Since all terms in, say, Eq. 10 have not the same sign (because points in layer i may be warmer as well as colder than surface m) we shall seek an estimate of the truncation error on the spatial density. For Eq. 10, the absolute value of this density reads :

$$\left|\frac{\partial\overline{\psi}^{sg}(m,y)}{\partial y}\right| = \sum_{r=0}^{\infty}\int_0^1 d\mu(0)...\int_0^1 d\mu(r)\left|\overline{h}^{sg}\Big(l,\Gamma(S(m),y,r,\vec{\mu},m')\Big)\right| \tag{14}$$

Clearly the partial sum of the r_0 first terms yields an under estimate of the full sum. What is sought for now is an overestimate function ξ^{sg} such that :

$$\int_0^1 d\mu(0)...\int_0^1 d\mu(r_0)\left|\sum_{r=r_0+1}^{\infty}\int_0^1 d\mu(r_0+1)...\int_0^1 d\mu(r)\ \overline{h}^{sg}\Big(l,\Gamma(S(m),y,r,\vec{\mu},m')\Big)\right|$$
$$\leq \int_0^1 d\mu(0)...\int_0^1 d\mu(r_0)\ \xi^{sg}\Big(l,\Gamma(S(m),y,r_0,\vec{\mu},m')\Big) \tag{15}$$

The domain of integration of the truncated sum and the one of the over estimate function ξ^{sg} are the same : the angles of the first r_0 terms. So these two numerical integrations can be computed together.

Simple over-estimate functions ξ^{sg} could be derived by only considering surface extinction after the (r_0+1)-th reflection. As an alternative, other over-estimates are proposed here for which only gas extinction is considered. The derivations are made with the assumption that the spectral average transmission function is a direct function of the effective pressure path length u_γ (common to most narrow band models):

$$\overline{F}_l(\gamma) = \overline{T}_l(u_\gamma) \tag{16}$$

u_γ is defined similarly to the $\tau_{\nu,\gamma}$ expression (Eq. 3):

$$u_\gamma = \begin{cases} u\Big(x_\gamma, y_\gamma, \mu_\gamma(0)\Big) & \text{if } r_\gamma = 0 \\ u\Big(x_\gamma, S(n_\gamma), \mu_\gamma(0)\Big) + \sum_{s=1}^{r_\gamma-1} u\Big(S(1), S(2), \mu_\gamma(s)\Big) + u\Big(S(M_\gamma(r_\gamma)), y_\gamma, \mu_\gamma(r_\gamma)\Big) & \text{if } r_\gamma \geq 1 \end{cases} \tag{17}$$

where $u(x,y,\mu)$ is the effective pressure path length of the segment from abscissa x to abscissa y with a cone angle cosine μ (see Cherkaoui et al., 1996). For any path γ we shall need extended path $\gamma|r$, for reflection numbers $r > r_\gamma$, defined as :

$$\gamma|r = \Gamma(x_\gamma, y_\gamma, r, \vec{\mu}_{\gamma|r}, n_\gamma) \tag{18}$$

where

$$\mu_{\gamma|r}(s) = \mu_\gamma(s) \qquad \text{if} \quad 0 \le s \le r_\gamma - 1 \tag{19}$$

$$\mu_{\gamma|r}(s) = \mu^* \qquad \text{if} \quad r_\gamma \le s \le r - 1 \tag{20}$$

$$\mu_{\gamma|r}(r) = \mu_\gamma(r_\gamma) \tag{21}$$

$\mu^* = 1$ in the general case and could be taken as $\mu^* = \mu_\gamma(0)$ in the simple case of two specular reflecting surfaces. $\gamma|r$ is therefore an optical path that is identical to γ up to the $(r_\gamma - 1)$-th reflection, other cone angle cosines being attributed the value μ^* except for the last direction that corresponds to the last direction of the path γ. A first over-estimate can be obtained by use of the properties that : i) directional-hemispheric reflectivities are lower than unity; ii) the first derivative of $\overline{T}$ (which is negative) is an increasing function of the effective pressure path length.

$$\begin{aligned} &\left| \sum_{r=r_0+1}^{\infty} \int_0^1 d\mu(0) ... \int_0^1 d\mu(r)\ \overline{h}^{sg}\Big(l, \Gamma(S(m), y, r, \vec{\mu}, m')\Big) \right| \\ &\quad \le \int_0^1 d\mu(0) ... \int_0^1 d\mu(r_0) \left\{ \pi\ \left[1 - \overline{\rho}_l^{dh}(\mu_\gamma(0), m)\right]\ \left|\overline{B_l}(y_\gamma) - \overline{B_l}(S(m))\right| \right. \\ &\qquad \left. \times\ 2\ \mu_\gamma(0)\ \overline{R}_{l,\gamma} \overline{\Xi}_{l,\gamma} \sum_{r=r_\gamma+1}^{\infty} \left[-\frac{\partial \overline{T}_l}{\partial u}\left(u_{\gamma|r}\right)\right] \right\}_{\gamma=\Gamma(S(m),y,r_0,\vec{\mu},m')} \end{aligned} \tag{22}$$

where

$$\overline{\Xi}_{l,\gamma} = \max_{\substack{\alpha \in [0,1] \\ i \in \{1,2\}}} \left(\frac{\overline{q}_l\left(\mu_\gamma(r_\gamma); \alpha, i\right)}{\overline{q}_l\left(\mu_\gamma(r_\gamma); \mu_\gamma(r_\gamma - 1), M_\gamma(r_\gamma)\right)} \right) \tag{23}$$

Note that $\overline{\Xi}_{l,\gamma} = 1$ for specular or diffuse reflecting surfaces. Further simplification of this over-estimate can be obtained by use of the following relation :

$$u_{\gamma|r+2t} = u_{\gamma|r} + 2\ t\ u\Big(S(1), S(2), \mu^*\Big) \tag{24}$$

together with the following property of positive monotonous decreasing functions :

$$\sum_{t=1}^{\infty} g(a+th) \leq \frac{1}{h} \int_{a}^{\infty} g(y)dy \tag{25}$$

In the present case, $g = -\frac{\partial \overline{T}_l}{\partial u}$ and $h = 2u(S(1), S(2), \mu^*)$ leads to :

$$\begin{aligned}
\sum_{r=r_\gamma+1}^{\infty} \left[-\frac{\partial \overline{T}_l}{\partial u}\left(u_{\gamma|r}\right)\right] &= -\frac{\partial \overline{T}_l}{\partial u}\left(u_{\gamma|r_\gamma+1}\right) - \sum_{t=1}^{\infty} \frac{\partial \overline{T}_l}{\partial u}\left(u_{\gamma|r_\gamma+2t}\right) - \sum_{t=1}^{\infty} \frac{\partial \overline{T}_l}{\partial u}\left(u_{\gamma|r_\gamma+1+2t}\right) \\
&\leq -\frac{\partial \overline{T}_l}{\partial u}\left(u_{\gamma|r_\gamma+1}\right) + \frac{\overline{T}_l(u_\gamma)}{2u(S(1), S(2), \mu^*)} + \frac{\overline{T}_l(u_{\gamma|r_\gamma+1})}{2u(S(1), S(2), \mu^*)} \\
&\leq -\frac{\partial \overline{T}_l}{\partial u}\left(u_{\gamma|r_\gamma+1}\right) + \frac{\overline{T}_l(u_\gamma)}{u(S(1), S(2), \mu^*)}
\end{aligned} \tag{26}$$

The final retained over-estimate (see Eq. 15, 22 and 26) is :

$$\begin{aligned}
\xi^{sg}\left(l,\gamma\right) &= \pi\left[1-\overline{\rho}_l^{dh}(\mu_\gamma(0), m)\right]\ \left|\overline{B_l}(y_\gamma) - \overline{B_l}(S(m))\right| \\
&\times 2\ \mu_\gamma(0)\ \overline{R}_{l,\gamma}\Xi_{l,\gamma}\left\{-\frac{\partial \overline{T}_l}{\partial u}(u_\gamma) + \frac{\overline{T}_l(u_\gamma)}{u(S(1), S(2), \mu^*)}\right\}
\end{aligned} \tag{27}$$

Gas-gas and surface-surface over-estimates are derived similarly :

$$\xi^{gg}\left(l,\gamma\right) = \pi\ \left|\overline{B_l}(y_\gamma) - \overline{B_l}(S(m))\right| 2\ \mu_\gamma(0)\ \overline{R}_{l,\gamma}\Xi_{l,\gamma}\left\{\frac{\partial^2 \overline{T}_l}{\partial u^2}(u_\gamma) - \frac{\frac{\partial \overline{T}_l}{\partial u}(u_\gamma)}{u(S(1), S(2), \mu^*)}\right\} \tag{28}$$

$$\begin{aligned}
\xi^{ss}\left(l,\gamma\right) &= \pi\left[1-\overline{\rho}_l^{dh}(\mu_\gamma(0), m)\right].\left[1-\overline{\rho}_l^{dh}(\mu_\gamma(r_\gamma), M_\gamma(r_\gamma))\right]\ \left|\overline{B_l}(S(1)) - \overline{B_l}(S(2))\right| \\
&\times 2\ \mu_\gamma(0)\ \overline{R}_{l,\gamma}\overline{\Xi}_{l,\gamma}\left\{\overline{T}_l\left(u_\gamma\right) + \frac{\displaystyle\int_0^{u_\gamma} \overline{T}_l\left(u'\right)du'}{u(S(1), S(2), \mu^*)}\right\}
\end{aligned} \tag{29}$$

For surface-surface exchanges, the primitive of $\overline{T}$ is difficult to handle. In Ref. (Cherkaoui et al., 1996), a simpler over-estimate was used that is valid in the specular and diffuse cases ; gas extinction is neglected after the (r_0+1)-th reflection and reflectivities are assumed either Dirac like or independent of the incident angle :

$$\begin{aligned}
\xi^{ss}\left(l,\gamma\right) &= \pi\left[1-\overline{\rho}_l^{dh}(\mu_\gamma(0), m)\right].\left[1-\overline{\rho}_l^{dh}(\mu_\gamma(r_\gamma), M_\gamma(r_\gamma))\right]\ \left|\overline{B_l}(S(1)) - \overline{B_l}(S(2))\right| \\
&\times 2\ \mu_\gamma(0)\ \overline{R}_{l,\gamma}\ \Xi_{l,\gamma}\ \overline{T}_l\left(u_\gamma\right).\frac{\overline{\rho}_l^{dh}(\mu^*,1).\overline{\rho}_l^{dh}(\mu^*,2)}{1-\overline{\rho}_l^{dh}(\mu^*,1).\overline{\rho}_l^{dh}(\mu^*,2)}
\end{aligned} \tag{30}$$

4. Test results

Preceding derivations allow the use of the EMCM algorithm (Cherkaoui at al., 1996) for configurations with large numbers of multiple reflections. Simulation results are presented for one-dimensional slab configurations with air-CO_2 mixtures and purely specular or purely diffuse surfaces. Spectral data are those of Soufiani et al.(1985) and Soufiani (1994).

A first series of simulations was made to validate the overestimation properties of the truncation error estimates derived in Sec. 3. The truncated volumetric radiation budget is an under estimate of the total budget and adding to it the truncation error over estimate yields an over estimate. Both under and over estimates have to converge when the number of considered reflections increases to infinity. Such a test is presented in Fig. 2 for a configuration with two perfect reflectors surfaces: convergence of the radiation budget is achieved without noticeable bias even for several hundred reflections. Also to be mentioned is the fact that the over estimate converges rapidly to the final solution; after a few reflections, the truncation error overestimate proposed in Sec. 3 provides a precise estimate of the sum of all radiative exchange via the infinite number of further reflections.

These tests were required to gain confidence in the EMCM as a method for production of reference solutions that may be useful for the design of approximate methods such as those presented in (Menart et al., 1993). An important information for this goal is for instance the number of multiple reflections to be taken into account for a given accuracy and a given configuration type. Let $\zeta(i,j;r)$ be the ratio of the NER between the i^{th} and the j^{th} gas layers via r reflections over the full NER $\psi(i,j)$. Since the ratio $\zeta(i,j;r)$ is, for highly reflective configurations and relatively thin media, approximately identical for all layer pairs, only its average value $\zeta(r)$ will be considered. Fig. 3 gives the evolution of $\zeta(r)$ versus the number of reflections for three dioxyde partial pressure P_{CO_2} values. For $P_{CO_2} = 10^{-4}$, 50% of the exchanges are due to the 10 first reflections; however it is interesting to note that 10% of the exchanges are occurring via more than 100 reflections and 1% via more than 800 reflections. Of course, the number of reflections decreases strongly as soon as the surface emissivities are not strictly zero : for emissivity values of 0.1 and $P_{CO_2} = 10^{-4}$, the

25 first reflections contribute to 99% of the average energy NER (Fig. 4).

Finally, some results are presented that illustrate the difference between the purely specular and purely diffuse assumptions. The ratio between the radiation budget for diffuse surfaces and the one for specular surfaces is plotted in Fig. 5 (for three specific layers) versus the partial pressure of CO_2. Along the reflective surface the ratio diffuse/specular starts at a value of one for extremely small concentrations of absorbing material and increases regularly with P_{CO_2}, up to 13% for pure carbon dioxide. For gas volumes farther from the surface, the sensitivity to the reflective properties is much smaller (a few percent). The surface radiation budget appears to be much less sensitive to the reflective properties, as it was previously observed by Nelson (1979) for an isothermal gas volume. The same kind of results were obtained in the case of two highly reflective surfaces ($\epsilon_1 = \epsilon_2 = 0.1$).

5. Conclusion

NEF and EMCM have been extended to configurations involving multiple reflection processes. Reference solutions are therefore available that should be very useful to test approximate methods in particular as far as spectral correlation problems are concerned. It is of interest to note that:

i. Specular and diffuse reflection assumptions yield very similar results for most configuration types : the multiple angular integration complexity can be avoided for first approximation computations.

ii. Truncation errors can be precisely estimated : in the specular case, such estimates take simple mathematical forms. Thus multiple reflection exchanges should be easy to model with better accuracy than the commonly encountered assumption of uncorrelated reflections.

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*List of Figures

1 This figure displays an optical path $\gamma = \Gamma(X_1, X_2, 4, \vec{\Upsilon}, 2)$, i.e. going from $x_\gamma = X_1$ to $y_\gamma = X_2$, leaving X_1 towards surface $n_\gamma = 2$, with $r_\gamma = 4$ reflections. The optical path is made of 5 straight segments, which yields a vector of cone angle cosines $\vec{\mu}_\gamma = \vec{\Upsilon}$ which has a dimension of 5 (only the first component $\Upsilon(0)$ is shown). . . 12

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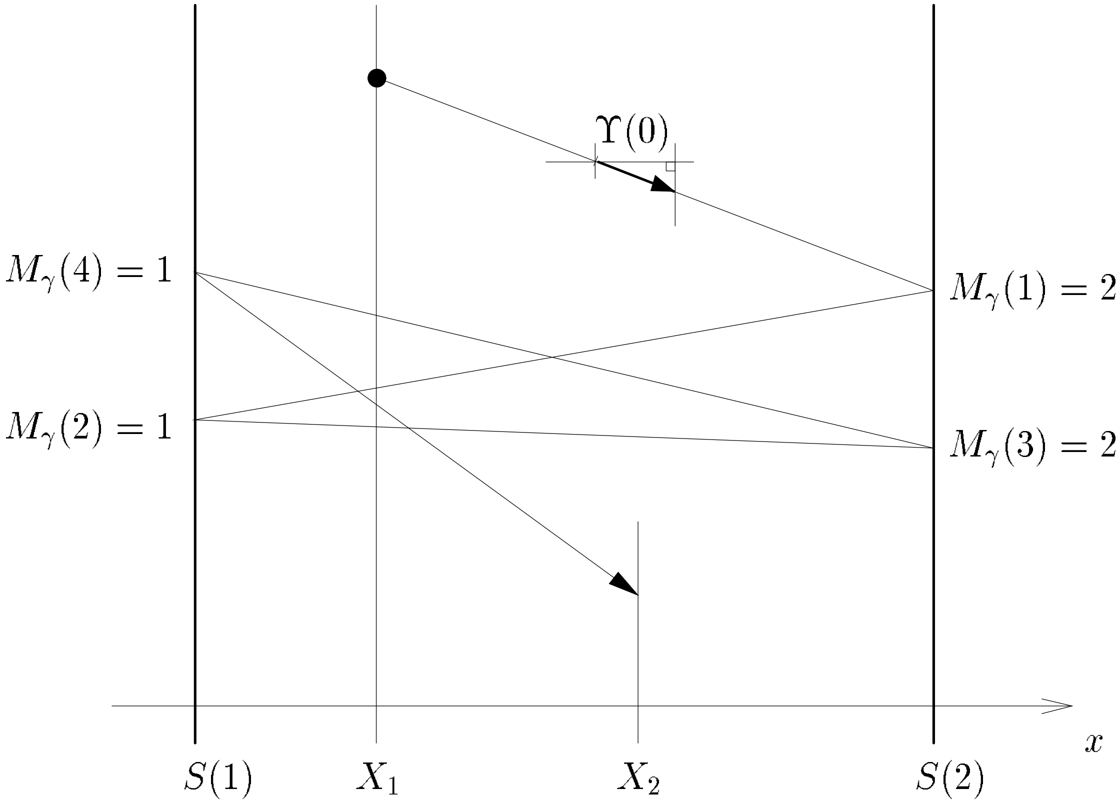
Figure 1: This figure displays an optical path $\gamma = \Gamma(X_1, X_2, 4, \vec{\Upsilon}, 2)$, i.e. going from $x_\gamma = X_1$ to $y_\gamma = X_2$, leaving X_1 towards surface $n_\gamma = 2$, with $r_\gamma = 4$ reflections. The optical path is made of 5 straight segments, which yields a vector of cone angle cosines $\vec{\mu}_\gamma = \vec{\Upsilon}$ which has a dimension of 5 (only the first component $\Upsilon(0)$ is shown).



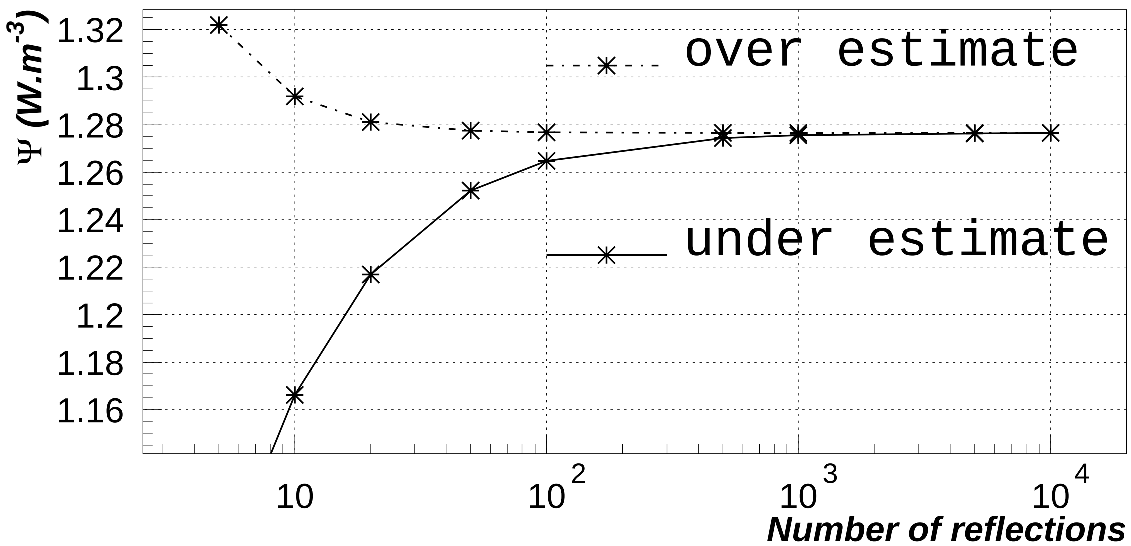


Figure 2: Under and over estimate of the radiation budget for a particular layer (located at $\approx 0.04m$ from $S(1)$) as a function of the number of reflection. $P_{CO_2} = 10^{-3}$, the two surfaces are perfect specular reflectors ($\epsilon_1 = \epsilon_2 = 0$), the temperature profile is linear ($295K$ at $S(1) = 0$ to $305K$ at $S(2) = 1m$).

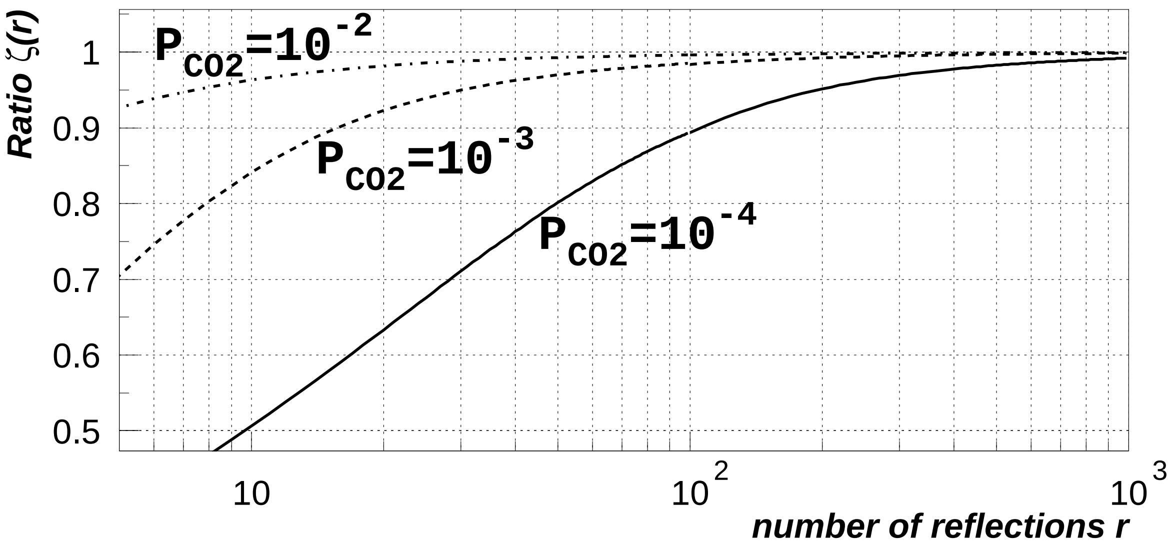


Figure 3: Ratio $\zeta(r)$ between the energy net-exchange rate through the r first reflections and the net-exchange rate through an infinite number of reflections as a function of r. The two surfaces are perfect diffuse reflectors ($\epsilon_1 = \epsilon_2 = 0$), the temperature profile is linear ($295K$ at $S(1) = 0$ to $305K$ at $S(2) = 1m$).

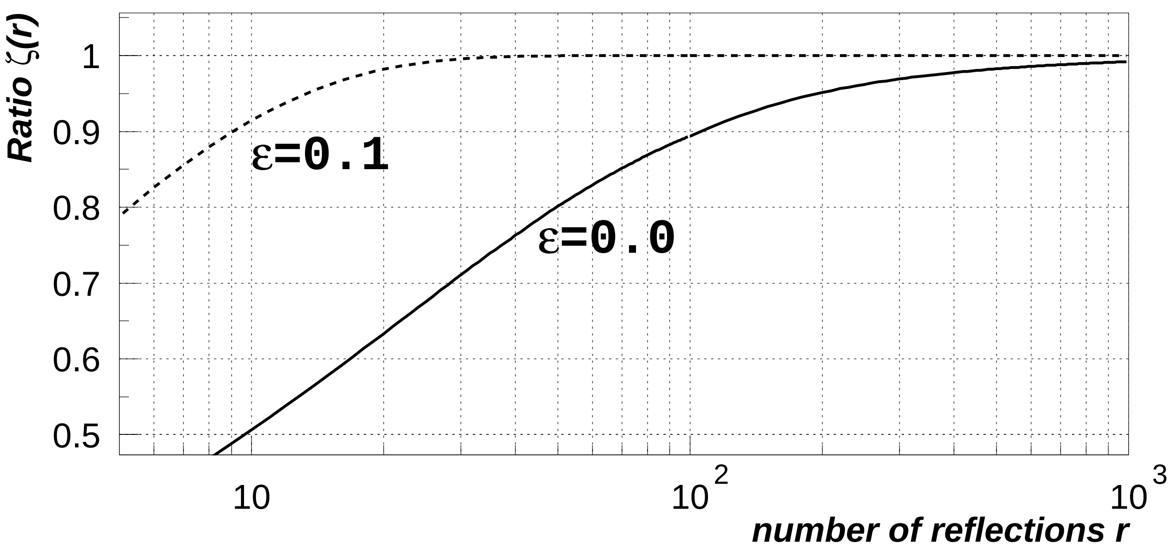


Figure 4: Same ratio $\zeta(r)$ as Fig. 3 but for two values of the surface reclecivity ($\epsilon_1 = \epsilon_2 = \epsilon$). The dioxyde partial pressure is $P_{CO_2} = 10^{-4}$.

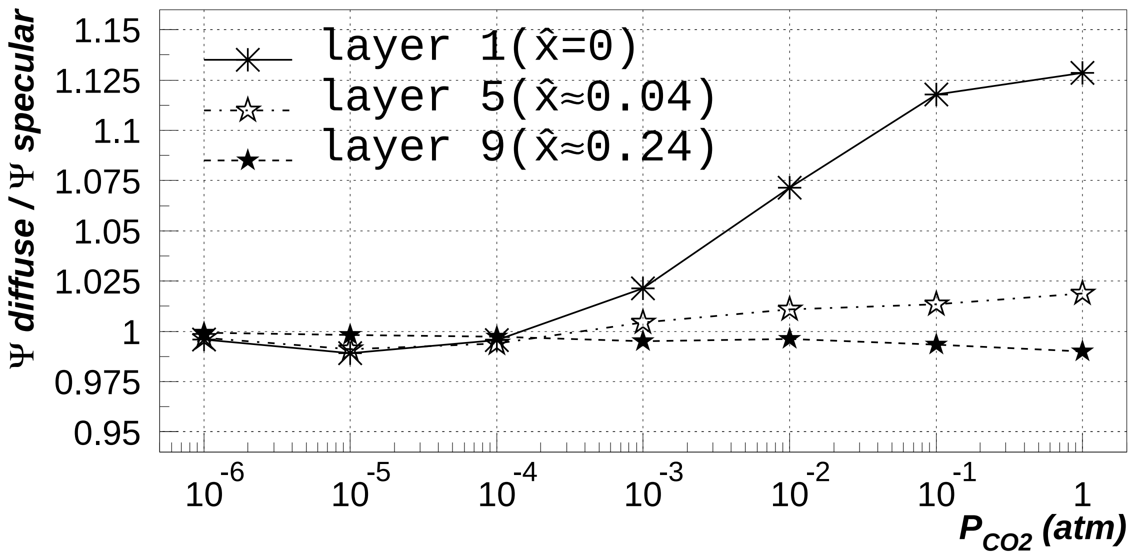


Figure 5: Ratio between the radiation budgets for diffuse reflections and the one for specular reflections versus partial pressures of absorbing gas (CO_2), in the case of one perfect reflector and one black surface ($\epsilon_1 = 0$, $\epsilon_2 = 1$). The temperature profile is linear ($295K$ at $S(1) = 0$ to $305K$ at $S(2) = 1m$). Layer 1 is zero thickness and located along the reflective surface, layer 5 and layer 9 are respectively located at 0.04m and at 0.24m .