Influence of Gravity Waves on the Atmospheric Climate

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1)Dynamical impact of mountains on atmospheric flows

2)Representations of mountains in General Circulation Models

3)Non-orographic gravity waves sources and breaking

4)Impact of gravity waves on the middle atmosphere dynamics

Influence of Gravity Waves on the Atmospheric Climate

3) Non-orographic gravity waves sources and breaking

   a) Hydrostatic waves equations and sources

   b) Wave-mean flow interaction

   c) Wave breaking parameterization

3)Non orographic gravity wave sources and breaking
a)Hydrostatic waves equations and sources

The Hydrostatic approximation, as the Boussinesq equations permits to filter sound waves.

Advantages:
In log-pressure coordinate the Hydrostatic equs look almost incompresible.
They easily permit to include the decrease with altitude of air density.

Defect:
Not suitable to describe trapped waves, KH instabilities...
Problem to represent the lower boundary

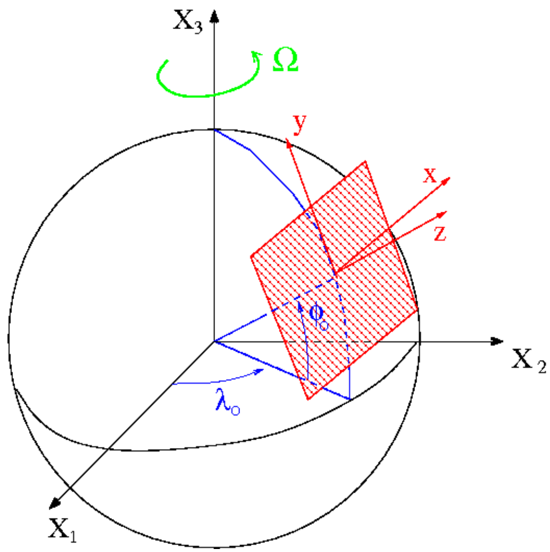
There has been a lot of research to find Eqs. that keep the advantages of both approximation, yielding to the anelastic equations.

Today, this will concerns more the theoretical studies, since more and more models treat the full compressible equations

3)Non orographic gravity wave sources and breaking
a)Hydrostatic waves equations and sources

Tangent f-plane geometry

No beta effect *(f=cte)*



Log pressure coordinate: $z = H \ln(\frac{p_r}{p})$

Charcateristic Height: $H = \frac{R T_r}{g} = 7\text{km}$ middle atmosphere representative

Density function: $\rho_0(z) = \rho_r \exp(-z/H)$

Equations:

$$\frac{Du}{Dt} - f v = -\frac{\partial \Phi}{\partial x} + G_x$$

$$\frac{Dv}{Dt} + f u = -\frac{\partial \Phi}{\partial y} + G_y$$

$$0 = -\frac{\partial \Phi}{\partial z} + \frac{R T}{H}$$

$$\partial_x u + \partial_y v + \frac{1}{\rho_0} \partial_z \rho_0 w = 0$$

$$\frac{D \Phi_z}{Dt} + \frac{\kappa \Phi_z}{H} w = J$$

Φ Is the potential,

$$\frac{D}{Dt} = \partial_t + u \partial_x + v \partial_y + w \partial z$$

and

$$w = \frac{Dz}{Dt}$$

3)Non orographic gravity wave sources and breaking
a)Hydrostatic waves equations and sources

Pure Gravity waves (3D, $N^2=cte,\ u_0=0,\ f=0$)

Linearized equations:

$$\partial_t u' + \partial_x \Phi' = 0$$
$$\partial_t v' + \partial_y \Phi' = 0$$
$$\partial_t \Phi'_z + N^2 w' = 0$$
$$\partial_x u' + \partial_y v' + \rho_0^{-1} \partial_z \rho_0 w' = O$$

Brunt Vaisala frequency:

$$N^2 = \Phi_{0zz} + \frac{\kappa}{H} \Phi_{0Z}$$

Looking for monochromatic solutions (the total solution can be reconstructed from them by Fourier series):

$$u' = \Re\left(\hat{u}\, e^{i(kx+ly+mz-\omega t)}\right) e^{z/2H}$$

We can take k>0 without lost of generality

Note the exponential growth with altitude of the solution

Dispersion relation

$$\omega = \pm N \sqrt{\frac{k^2+l^2}{m^2+\frac{1}{4H^2}}}$$

Phase velocity

$$\vec{c} = \frac{\omega}{|\vec{k}|^2}\begin{pmatrix} k & l & m \end{pmatrix}$$

Group velocity

$$\vec{c}_g = \vec{\nabla}_{\vec{l}}\, \omega = \omega \begin{pmatrix} \frac{k}{k^2+l^2} & \frac{l}{k^2+l^2} & \frac{-m}{m^2+1/(4H^2)} \end{pmatrix}$$

Remarks $\vec{C}$ and $\vec{C_g}$ are almost perpendicular:

Upward propagation:

$C_{gz}>0$ implies $m\omega<0$

3)Non orographic gravity wave sources and breaking
a)Hydrostatic waves equations and sources

Pure Inertio Gravity waves (3D, $N^2=cte$, $u_0=0$, $f\#0$)

Linearized equations:

$$\partial_t u' - f v' + \partial_x \Phi' = 0$$
$$\partial_t v' + f u' + \partial_y \Phi' = 0$$
$$\partial_t \Phi'_z + N^2 w' = 0$$
$$\partial_x u' + \partial_y v' + \rho_0^{-1} \partial_z \rho_0 w' = O$$

Dispersion relation

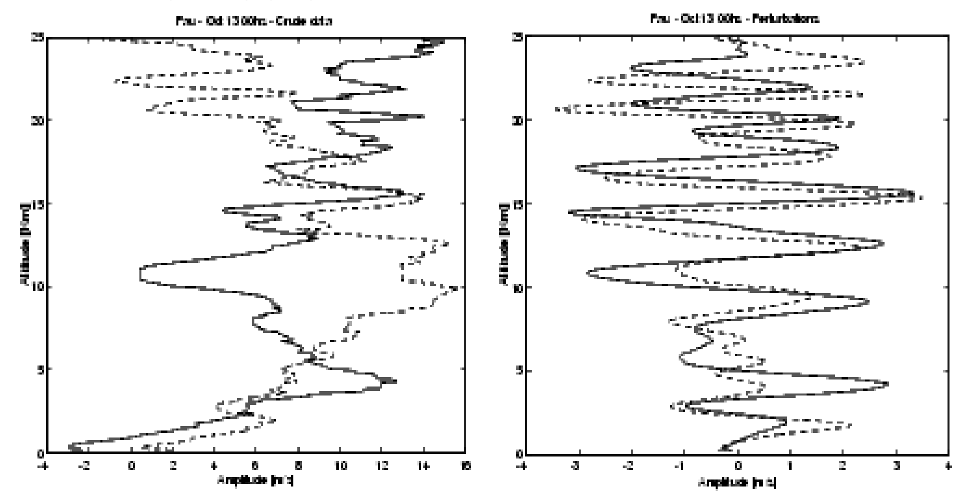
$$m^2 = \frac{N^2 (k^2 + l^2)}{\omega^2 - f^2} - \frac{1}{4H^2}$$

Only the waves with $\omega > f$ propagates vertically

Source 1 (fronts):

High res soundings somewhere above a front! Geostrophic Adjustment?

Raw data / filtered



For $\omega \sim f$, $v \sim -iu$, (both are in quadrature)

$m\omega<0$ (upward propagation) makes that u (solid) is in advance on v (dashed)

3)Non orographic gravity wave sources and breaking
a)Hydrostatic waves equations and sources

Source 1 (fronts):

Fundamental difficulty: dynamical separation between balance and GW (*slow quasi-manifold)*

Idealized numerical studies

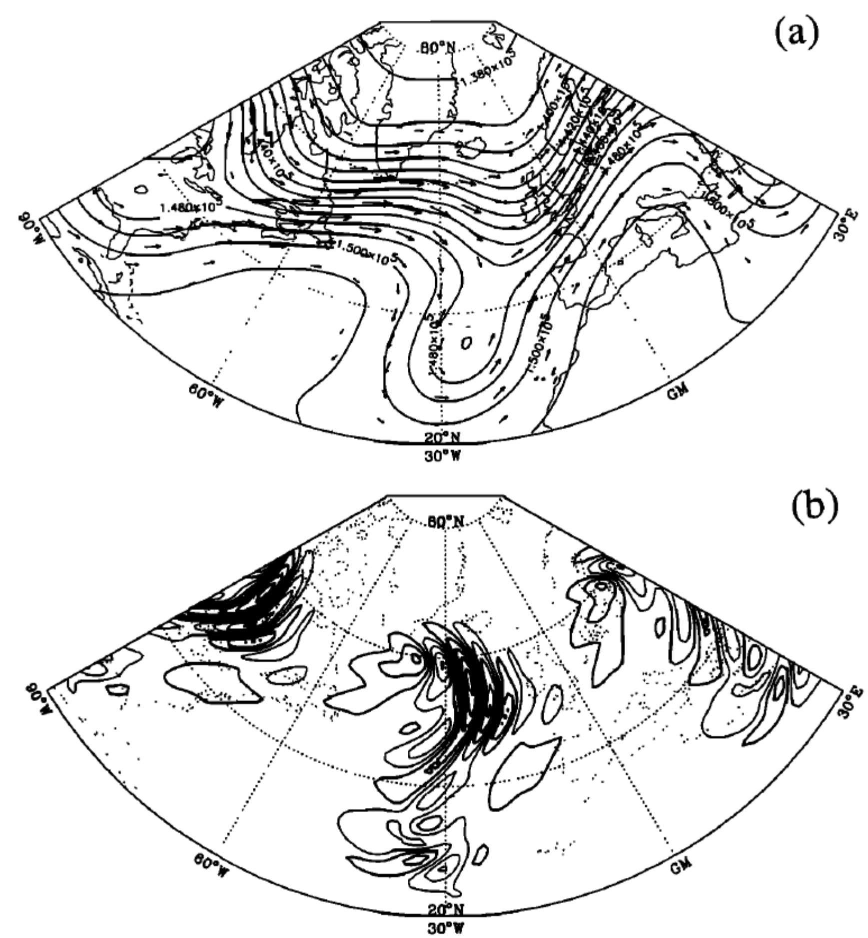
- 2D frontogenesis

  Ley and Peltier 78, Gall et al 88, Gardner 89, Snyder et al 93, Reeder and Griffiths 96, Griffiths and Reeder 96

- 3D baroclinic life cycles

  Van Tuyl and Young 82, **O'Sullivan & Dunkerton 95,** Bush, McWilliams & Peltier 95, Zhang 04, Viudez and Dritschel 06

→ **Small-scale** waves in Jet exit region



3)Non orographic gravity wave sources and breaking
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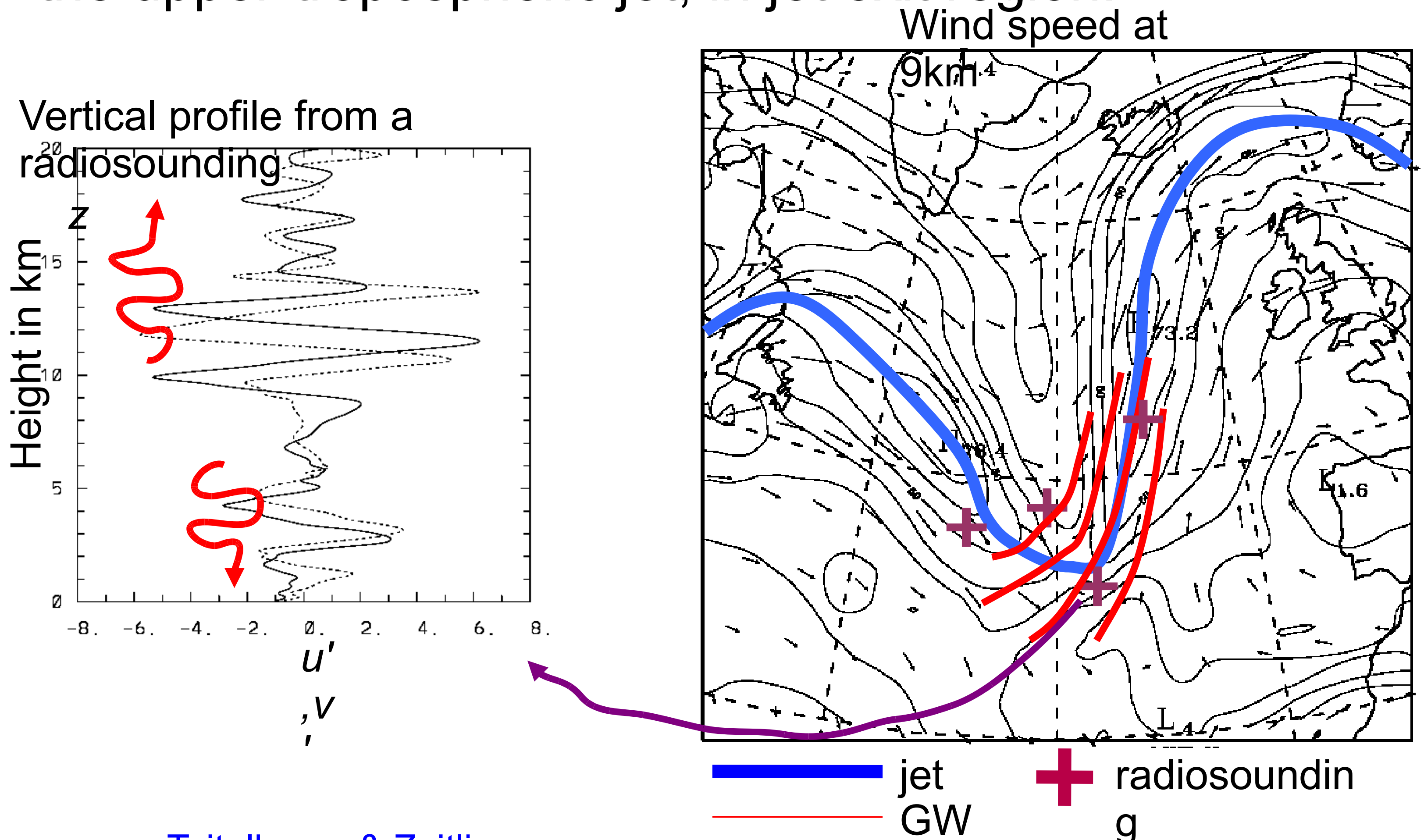
Source 1 (fronts):

- Low frequency, large amplitude wave emitted from the upper-tropospheric jet, in jet exit region:



Plougonven, Teitelbaum & Zeitlin 03

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Source 2: Mountains

Boundary condition

$$w' = \vec{u}(0) \cdot \vec{\nabla} h$$

As mountains impose ω=0, it is the presence of an incident wind that permits an oscillatory behaviour.

It also permits vertical propagation:

$$m^2 = \frac{N^2 k^2}{\hat{\omega}^2} - \frac{1}{4H^2}$$ (f=0 for simplicity)

The intrinsic frequency is:

$$\hat{\omega} = \omega - \vec{k} \cdot \vec{u}_0$$

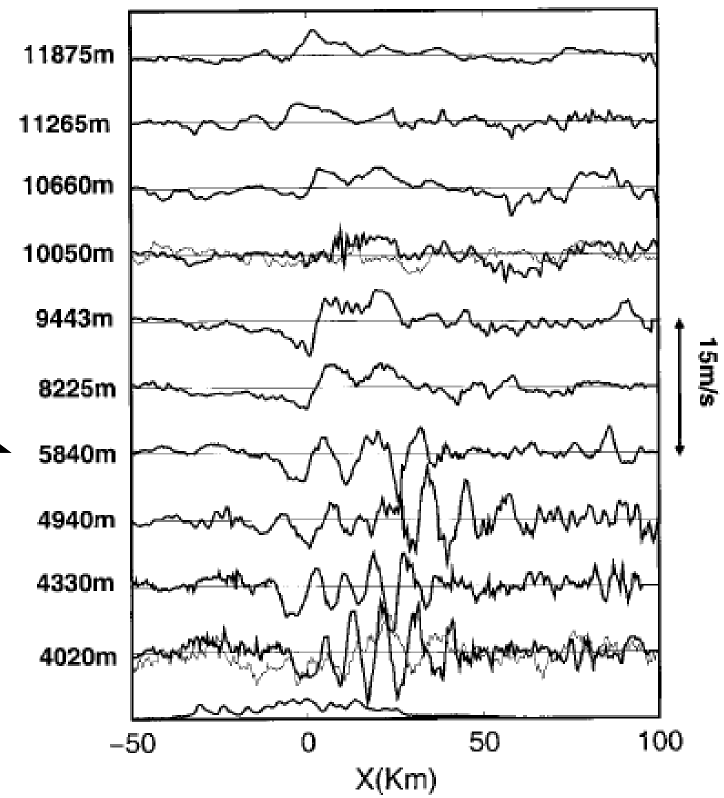

FIG. 2. Observed vertical velocities from different aircraft legs, from 15 Oct 1990 around 0600 UTC. Thick lower curve represents the Pyrénées; the thin curve at the Z = 4 km and Z = 10 km are red-noise surrogates with characteristics adapted to the measured vertical velocity at that level.

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Source 3: Convection

A diabatic heating stationnary in space
but fluctuating in time produces non-stationnary
waves in different directions of propagation.

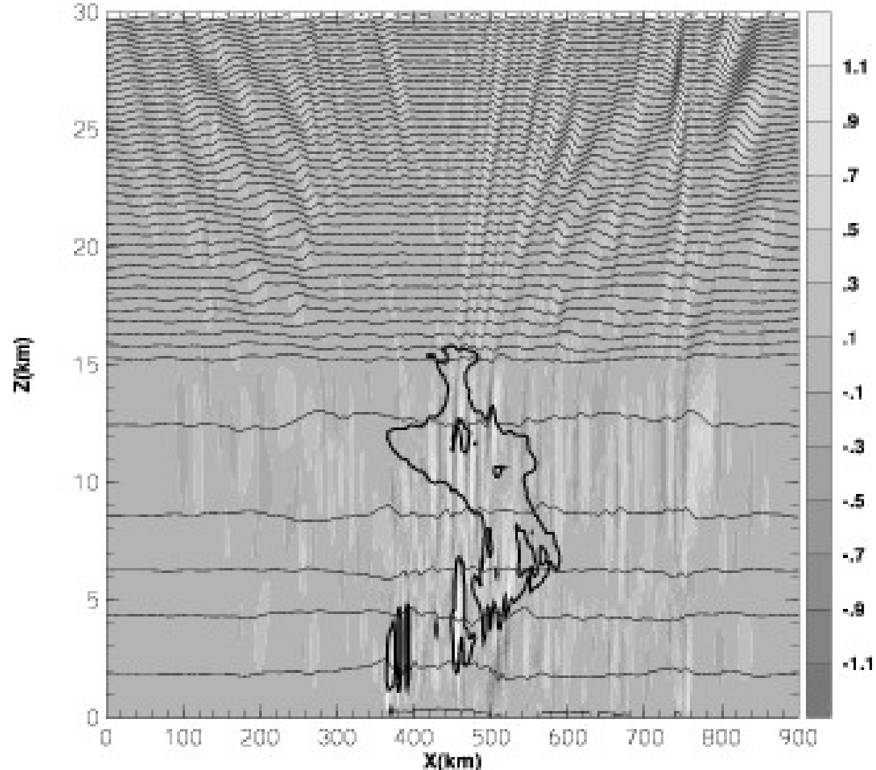
Very simple Heuristic example of heating at a
Given altitude:

$$J(x,t)=f(x)g(t)$$

If: $$f(x)=J_0 \cos kx\,,\; g(t)=\cos\omega t$$

$$J(x,t)=\frac{J_0}{2}\cos(kx-\omega t)+\frac{J_0}{2}\cos(kx+\omega t)$$

The heating produce waves in both directions
of propagation



Gravity waves above
a convective cloud
(Alexander et Holton 1997)

3)Non orographic gravity wave sources and breaking
b)Wave-mean flow interactions

Wave-mean flow separation in the 2D-periodic case
(again a purely formal approximation used here that facilitates the math.
The domain can be a model gridbox, and we assume that the waves stay in it)

$$u(x,z,t)=\bar{u}(z,t)+u'(x,z,t)$$
$$w(x,z,t)=w'(x,z,t)$$
$$T(x,z,t)=\bar{T}(z,t)+T'(x,z,t)$$

Remember:
$$\Phi_z=\frac{RT}{H}$$
$$\bar{a}=\frac{1}{2X}\int_{-X}^{+X} a\,dx$$

Mean flow equations (2d order):

$$\frac{\partial}{\partial t}\bar{u}=\frac{1}{\rho_0}\frac{\partial}{\partial z}-\rho_0\overline{u'w'}+\bar{G}_x$$
$$\frac{\partial}{\partial t}\bar{\Phi}_z=\frac{1}{\rho_O}\frac{\partial}{\partial z}-\rho_0\overline{w'\Phi'_z}+\bar{J}$$

The vertical component of the EP-flux

$$\overline{F^z}=-\rho_O\overline{u'w'}$$

Wave equations (1rst order)
(no breaking, no mechanical or thermal dissipation no thermal forcing).

$$\left(\partial_t+\bar{u}\,\partial_x\right)u'+\bar{u}_z w'=-\partial_x\Phi'$$
$$\left(\partial_t+\bar{u}\,\partial_x\right)\Phi'_z+\bar{N}^2 w'=0$$
$$\frac{\partial}{\partial x}u'+\frac{1}{\rho_0}\frac{\partial}{\partial z}\rho_0 w'=0$$

BV frequency:
$$\bar{N}^2=\bar{\Phi}_{zz}+\frac{\kappa}{H}\bar{\Phi}_z$$

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Vertical structure of a monochromatic wave
(remember we can return to the full disturbance via Fourier transforms)

$$\begin{pmatrix} u' \\ w' \\ \Phi' \end{pmatrix} = \Re\left[\begin{pmatrix} \hat{u}(z) \\ \hat{w}(z) \\ \hat{\Phi}(z) \end{pmatrix} e^{i(kx-\omega t)}\right] e^{z/2H} \longrightarrow$$

$$-i\hat{\omega}\hat{u} + \bar{u}_z \hat{w} = -ik\hat{\Phi}$$

$$-i\hat{\omega}\left(\hat{\Phi}_z + \frac{\hat{\Phi}}{2H}\right) + \bar{N}^2 \hat{w} = 0$$

$$ik\hat{u} + \left(\hat{w}_z - \frac{\hat{w}}{2H}\right) = 0$$

k>0 by convention

Intrinsic frequency: $\hat{\omega} = \omega - k\bar{u}$

Vertical structure equation:

$$\frac{d^2\hat{w}}{dz^2} + \underbrace{\left(\frac{\bar{N}^2 k^2}{\hat{\omega}^2} + \frac{k}{\hat{\omega}}\left(\bar{u}_{zz} + \frac{\bar{u}_z}{H}\right) - \frac{1}{4H^2}\right)}_{Q(z)}\hat{w} = 0$$

Remember, we want to evaluate the vertical component of the Eliassen Palm Flux:

$$\overline{F^z} = -\rho_O \overline{u'w'} = -\frac{\rho_r}{2}\Re\left[\hat{u}\,\hat{w}^{\dagger}\right]$$

3)Non orographic gravity wave sources and breaking
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Non-interarction theorem

$$\overline{F^z} = -\frac{\rho_r}{2}\Re\left[\hat{u}\,\hat{w}^{\dagger}\right] = \Re\left[-i\,\frac{\rho_r}{2k}\,\hat{w}_z\,\hat{w}^{\dagger}\right]$$

$$\int_{z_1}^{z_2}\left[\left(\frac{d^2\hat{w}}{dz^2}+Q(z)\,\hat{w}=0\right)x-\frac{i\,\rho_r}{k}\,\hat{w}^{\dagger}\right]dz \quad \xrightarrow{\text{IP}} \quad \left[-i\,\frac{\rho_r}{2k}\,\hat{w}_z\,\hat{w}^{\dagger}\right]_{z_1}^{z_2} = i\,\frac{\rho_r}{2k}\int_{z_1}^{z_2}\left[\hat{w}_z\,\hat{w}_z^{\dagger}+Q(z)\,\hat{w}\,\hat{w}^{\dagger}\right]dz$$

Pure imaginary, real part 0

$$\left[\overline{F^z}\right]_{z_1}^{z_2} = 0$$

For linear steady adiabatic non dissipative waves
This the way Eliassen and Palm first derived it in 1961!

3)Non orographic gravity wave sources and breaking
c)Wave breaking parameterization

This is one manner to treat this problem, but the important think in the following is to understand the central rôle that plays the non-interaction theorem

WKB Solution: the mean flow varies slowly in the vertical direction compared to the vertical wavelength of the waves.

We search for solutions of the form (assuming z=0 to be the level of the source):

$$\hat{w}(z) = W(z)\, e^{i\int_0^z m(z')dz'}$$

Slowly varying means: $|W_z| \ll |m\,W|,\ |m_z| \ll m^2,\ ect....$

$$\frac{d^2\hat{w}}{dz^2} + Q(z)\hat{w} = 0 \longrightarrow \underbrace{\frac{d^2W}{dz^2}}_{\text{2d order}} + \underbrace{2\mathrm{im}\frac{dW}{dz} + i\frac{dm}{dz}W}_{\text{1rst order}} + \underbrace{\left(Q(z) - m^2(z)\right)W}_{\text{leading order}} = 0$$

Leading Order: $m(z) = -sign(\hat{\omega})\sqrt{Q(z)}$ — The minus sign ensure upward propagation

First Order: $W = \hat{w}(0)\sqrt{\frac{m(0)}{m(z)}}$

$$\hat{w} = \hat{w}(0)\sqrt{\frac{m(0)}{m(z)}}\, e^{i\int_0^z m(z')dz'}$$

3)Non orographic gravity wave sources and breaking
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$$m(z) = -sign(\hat{\omega})\sqrt{Q(z)} \qquad \hat{w} = \hat{w}(0)\sqrt{\frac{m(0)}{m(z)}}\, e^{i\int_0^z m(z')dz'}$$

If we inject the WKB solution into the EPF definition:

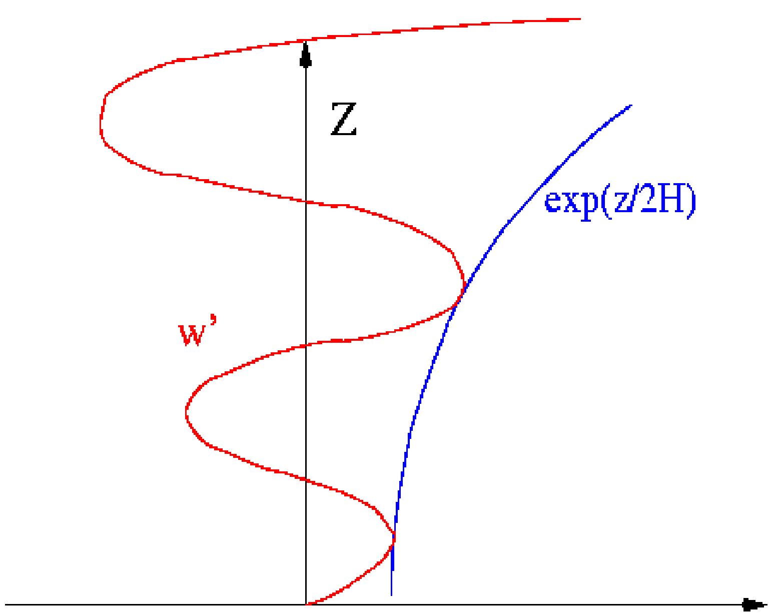
$$\overline{F}^z = -\frac{\rho_r}{2}\Re\left[\hat{u}\,\hat{w}^{\dagger}\right] = \Re\left[-i\frac{\rho_r}{2k}\hat{w}_z\,\hat{w}^{\dagger}\right] = \rho_r\frac{m(0)}{2k}\hat{w}(0)\,\hat{w}(0)^{\dagger} = cte$$

The WKB approximation satisfies the non-interaction theorem!

$$sign\left(\overline{F}^z\right) = -sign(\hat{\omega})$$

m(z) and *m(0)* always have the same sign otherwise $\hat{\omega}$ changes sign: there is a critical level where the wave breaks

Example for U=cte, N2=cte:



3)Non orographic gravity wave sources and breaking
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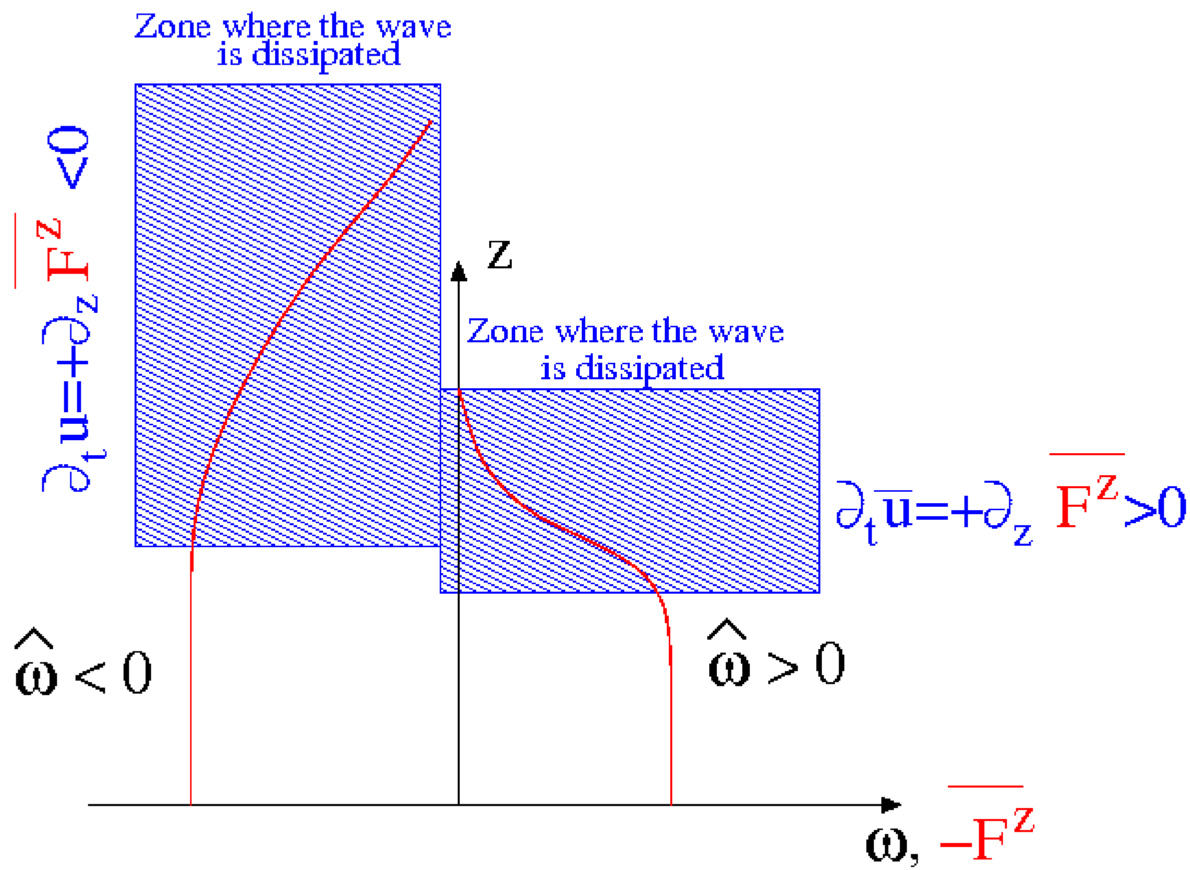
$$m(z) = -sign(\hat{\omega})\sqrt{Q(z)} \longrightarrow \boxed{sign(\bar{F}^z) = -sign(\hat{\omega})}$$

The sign chosen for m ensures upward propagation

It impose the sign of the vertical EP flux, with

Estward waves $\hat{c} = \hat{\omega}/k > 0$ accelerate the flow when they breack

Westward waves $\hat{c} = \hat{\omega}/k < 0$ decelerates the flow when they breack



3)Non orographic gravity wave sources and breaking
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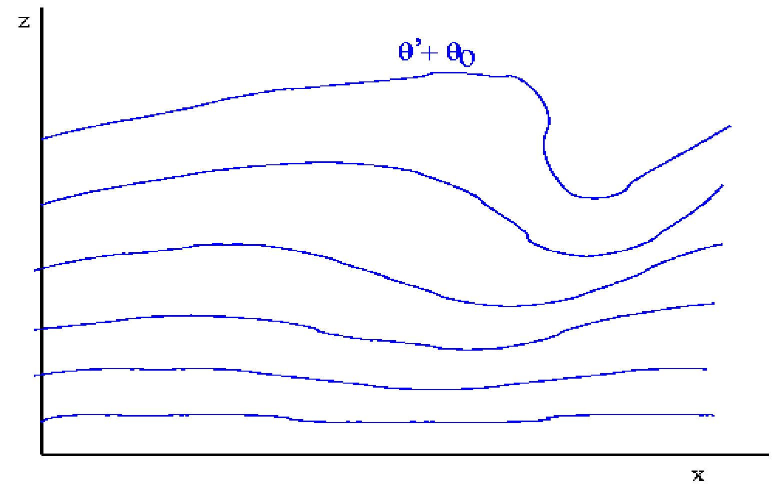
Breaking!

$$\frac{d(\theta'+\bar{\theta})}{dz}<0$$



Becomes in Hydrostatic-log pressure

$$\left|\hat{\Phi}_{zz}+\frac{\kappa}{H}\hat{\Phi}_z\right|e^{z/2H}>N^2$$

When using the polarization relations,
this becomes:

$$|\hat{w}|<\left|\frac{\hat{\omega}}{m}\right|e^{-z/2H}=w_s(z)$$

When using the WKB solution this translates

In term of stress:

$$\left|\overline{F^z}\right|<\left|\overline{F^z}_S\right| \quad \text{where} \quad \overline{F^z}_S=\frac{-\rho_r}{2k^2N}\hat{\omega}^3 e^{-z/2H}$$

No flux across critical levels!

For a constant flow, we can also evaluate a breaking altitude: $Zbr=2H\ln\left(\frac{\hat{\omega}^2}{Nk|\hat{w}(0)|}\right)$

Breaking over more rapidly in the vertical for large amplitude and small intrinsic frequency!

3)Non orographic gravity wave sources and breaking
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Waves breaking parametrization

$$\rho_0 \frac{\partial \bar{u}}{\partial t} + ... = \sum_{i=1}^{N} \frac{\partial \overline{F}_i^z}{\partial z}$$

We can put as many waves as one wants!

Wave breaking In term of stress:

No breaking if $\left|\overline{F^z}\right| < \left|\overline{F^z}_S\right|$ where $\overline{F^z}_S = \frac{-\rho_r}{2k^2 N} \hat{\omega}^3 e^{-z/2\mathrm{H}}$

1)By cleaver physival considerations, we impose at the source (z=0 here but any level works)

$$\overline{F^z}(0), \omega, k$$

z+dz

Model levels

z

2) Passage from z to z+dz

$$\overline{F^z}(z+dz) = \overline{F^z}(z)$$

$$if \left|\overline{F^z}(z+dz)\right| > \left|\overline{F^z}_S(z+dz)\right|$$

$$then: \overline{F^z}(z+dz) = \overline{F^z}_S(z+dz)$$

3)Non orographic gravity wave sources and breaking

Summary