Atmospheric Circulation

(WAPE: General Circulation of the Atmosphere and Variability)

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8) Stratospheric equatorial variability

a) Observations

Equatorial waves and quasi biennal oscillation

Refresh on Eq. Wave theory,

Level 1 statistics: time longitude spectra and composite

b) Quasi-Biennal Oscillation explained in terms of gravity waves mean flow interactions

Toy model 5

c) Returning to the Temperature minimum at the summer polar mesopause

a) Observations: Equatorial waves and quasi biennal oscillation

Refresh on Equatorial wave theory (Lecture 7 and TD 7)

For $v'=0$: Kelvin waves, see Lecture 7

Linear disturbances in an equatorial beta-plane
At rest (cte N^2, also here)
See Lecture 7 (mainly for Kelvin wave, $v'=0$)
But with Hydrostatic log pressure coordinates (as in TD7)

$$\partial_t u' - \beta y v' + \partial_x \Phi' = 0$$

$$\partial_t v' + \beta y u' + \partial_y \Phi' = 0$$

$$\partial_x u' + \partial_y v' + \rho_0^{-1} \partial_z \rho_0 w' = 0$$

$$\partial_t \partial_z \Phi' + N^2 w' = 0$$

$$\begin{pmatrix} u' \\ v' \\ \Phi' \\ w' \end{pmatrix} = \begin{pmatrix} \hat{u}(y) \\ \hat{v}(y) \\ \hat{\Phi}(y) \\ \hat{w}(y) \end{pmatrix} e^{z/2H} e^{i(kx+mz-\omega t)}$$

$$-i\omega \hat{u} - \beta y \hat{v} + ik\hat{\Phi} = 0$$

$$-i\omega \hat{v} + \beta y \hat{u} + \partial_y \hat{\Phi} = 0$$

$$-i\omega \hat{\Phi} + gh\left(ik\hat{u} + \partial_y \hat{v}\right) = 0$$

$$gh = c^2 = \frac{N^2}{m^2 + \frac{1}{4H^2}}$$

$\hat{v} \neq 0$

Meridional structure equation :

$$\partial_y^2 \hat{v} + \left(\frac{\omega^2}{c^2} - k^2 - \frac{\beta k}{\omega} - \frac{\beta^2 y^2}{c^2}\right)\hat{v} = 0$$

a) Observations: Equatorial waves and quasi biennal oscillation

Refresh on Equatorial wave theory (Lecture 7 and TD 7)

Solution for $\hat{v} \neq 0$ (For Kelvin waves, $\hat{v} = 0$, see Course 7)

Meridional structure equation :

$$\partial_y^2 \hat{v} + \left(\frac{\omega^2}{c^2} - k^2 - \frac{\beta k}{\omega} - \frac{\beta^2 y^2}{c^2} \right) \hat{v} = 0$$

Solutions
(Shrodinger Eqs. Harmonic oscillator)

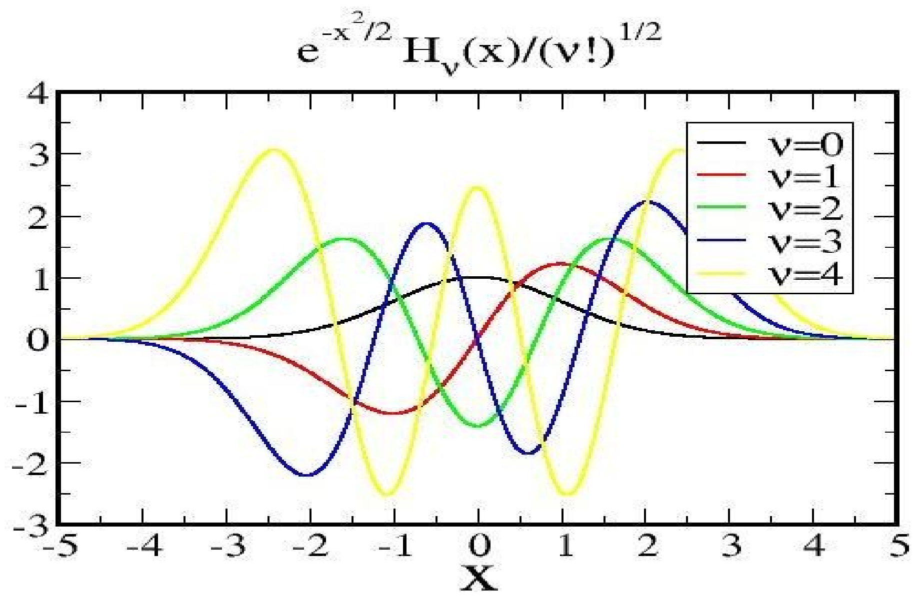
$$\hat{v} = e^{-\frac{\beta}{c}\frac{y^2}{2}} H_\nu \left(\left(\frac{\beta}{c} \right)^{1/2} y \right)$$

Hermitte polynomial

$$H_0 = 1, \quad H_1 = 2x, \quad H_2 = 4x^2 - 2, \quad ....$$

Dispersion relation when ν is given :

$$\frac{\omega^2}{c^2} - k^2 - \frac{k\beta}{\omega} = (2\nu + 1)\frac{\beta}{c}$$



$$gh = c^2 = \frac{N^2}{m^2 + \frac{1}{4H^2}}$$

ν : number of zero for $\hat{v}$ between the poles

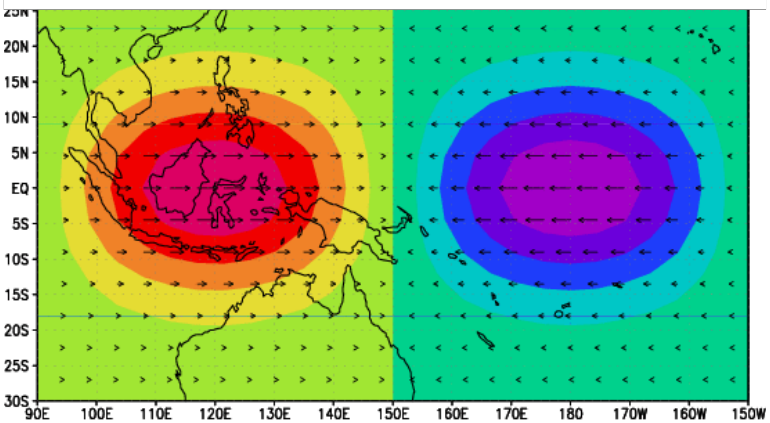
$\nu = -1$ for Kelvin waves

a) Observations: Equatorial waves and quasi biennal oscillation

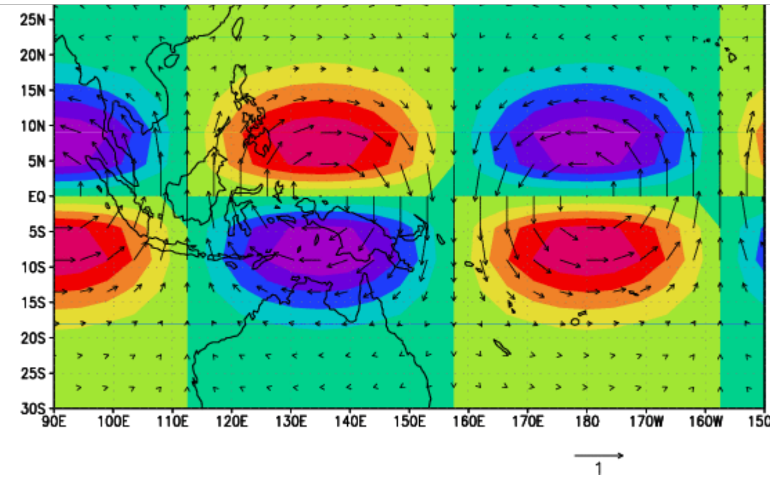
Refresh on Equatorial wave theory

Dispersion relation of the equatorially trapped, Kelvin waves (ν=-1), Rossby Gravity waves (ν=0), and Rossby waves (with ν=1), ect.
ω>0 convention, fields of horizontal wind and Φ:

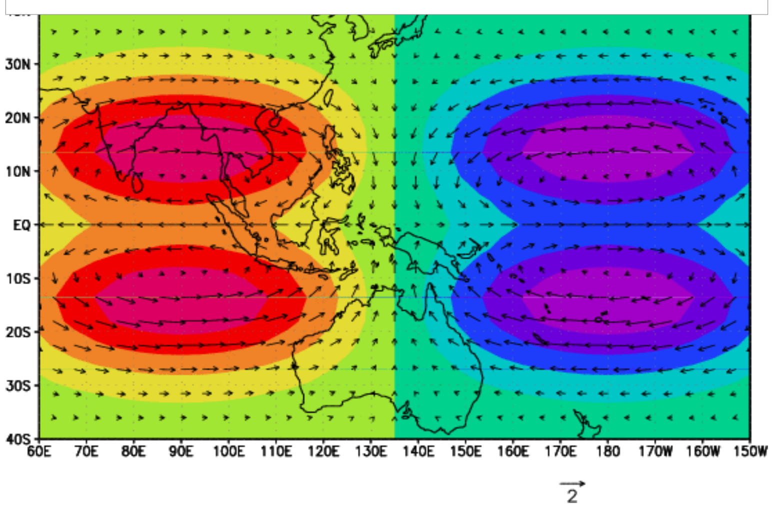
Kelvin wave: $\omega = kc$



Rossby-gravity: $\omega = \frac{\sqrt{k^2 c^2 + 4\beta c} + kc}{2}$



Rossby ν=1



Each ν>0 : 1 Rossby and 2 inertio-gravity

$$\frac{\omega^2}{c^2} - k^2 - \frac{k\beta}{\omega} = (2\nu+1)\frac{\beta}{c}$$

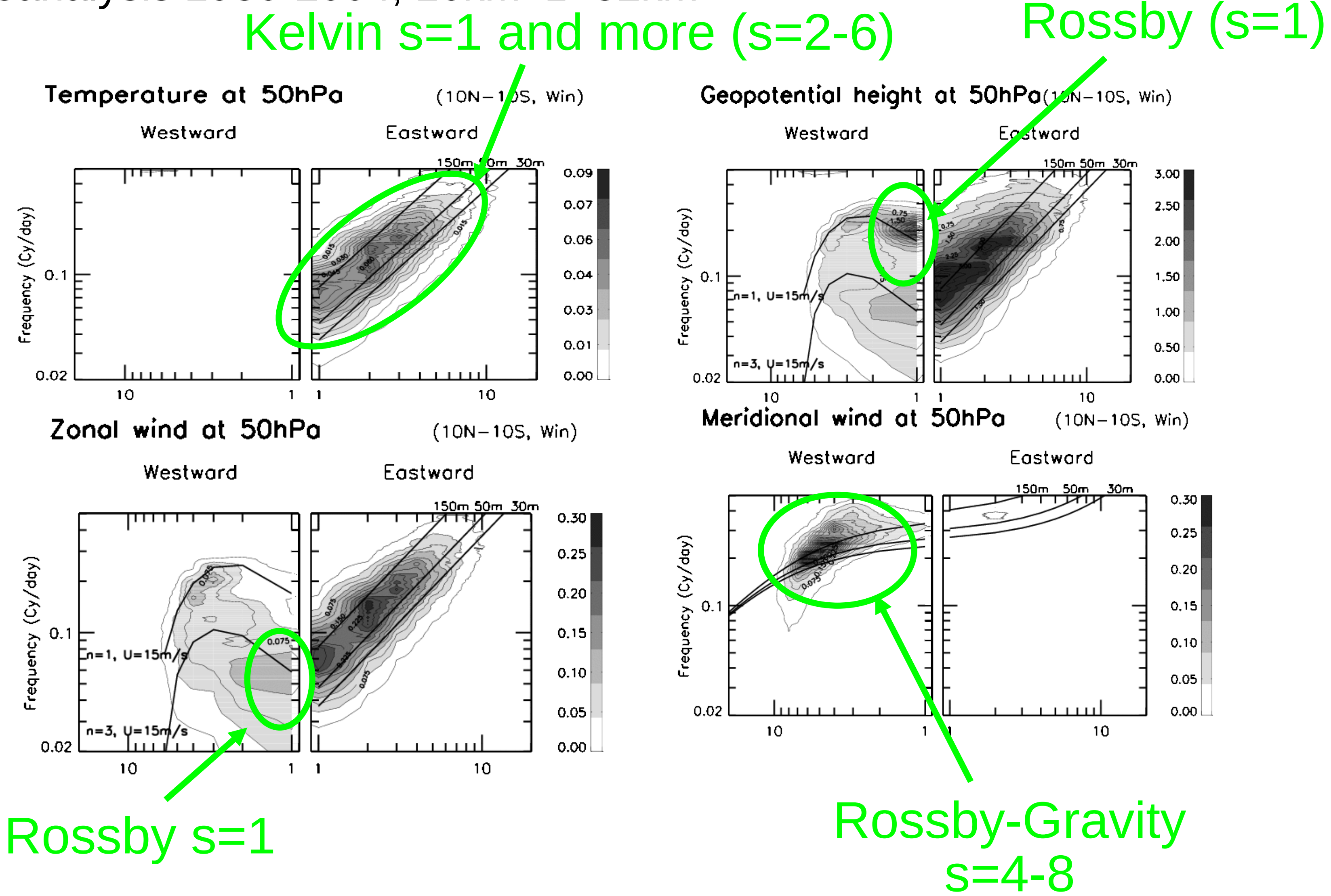
$$gh = c^2 = \frac{N^2}{m^2 + \frac{1}{4H^2}}$$

a) Observations: Equatorial waves and quasi biennal oscillation

Time-latitude Spectra and composites

Spectra in frequency ($\sigma=\omega/\Omega$) and wavenumber (s=ka) of the u, v, T, and Φ fields averaged over [10°S,10°N]

NCEP-Reanalysis 1980-2004, 16km<z<32km

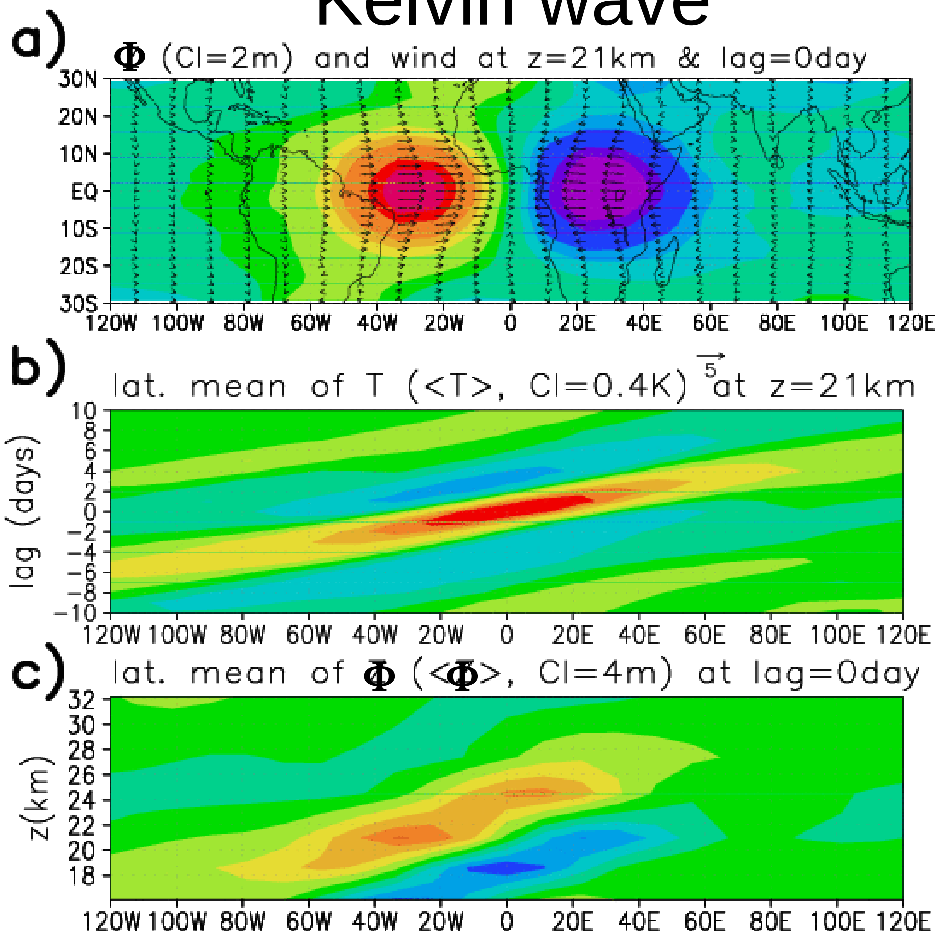


a) Observations: Equatorial waves and quasi biennal oscillation

Time-latitude Spectra and composites

Composites of Kelvin waves

Kelvin wave



Index for entrance in the low stratosphere based on T at 21km filtred in the band s=2-6, 1-10day periods.

Characteristic structure of a Kelvin wave

The amplitude is substantial (compare to the Standard Deviation of the unfiltered fields)

Eastward propagation, probably eastward Group speed.

Phase lines inclined eastward when altitude increases indicating upward propagation

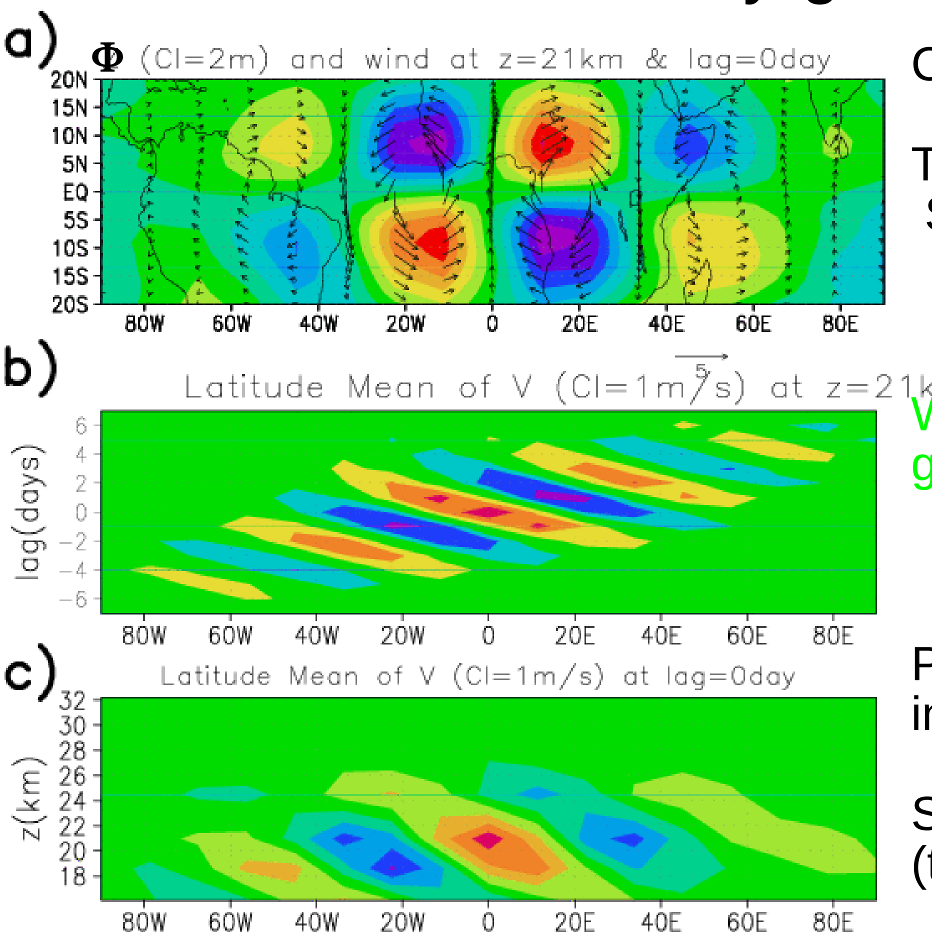
Signal attenuation in the vertical (through interaction with the QBO?)

a) Observations: Equatorial waves and quasi biennal oscillation

Time-latitude Spectra and composites

Composites of Rossby-gravity waves

The Rossby-gravity waves



Characteristic structure of a Rossby-gravity wave

The amplitude is substantial (compare to the Standard deviation of the unfiltered field)

Westward phase propagation but eastward group propagation

Phase lines inclined westward when altitude increases indicating upward propagation.

Signal attenuation in the vertical (through interaction with the QBO?)

The Quasi-Biennial Oscillation (low stratosphere)

Satellites wind observations (UARS, Swinbak et Ortland 1997)

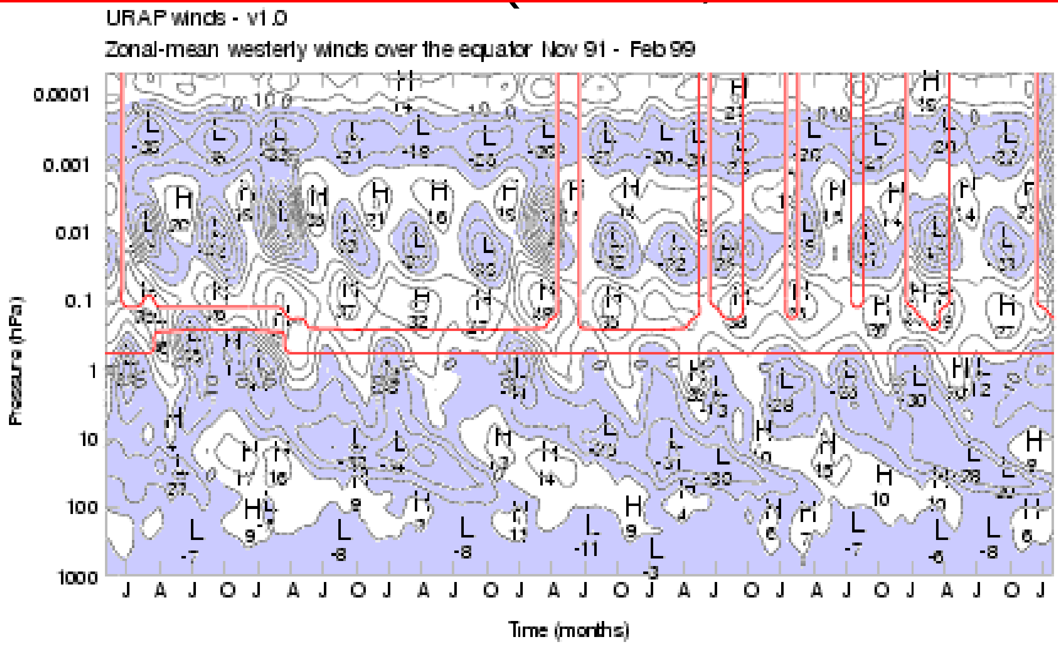
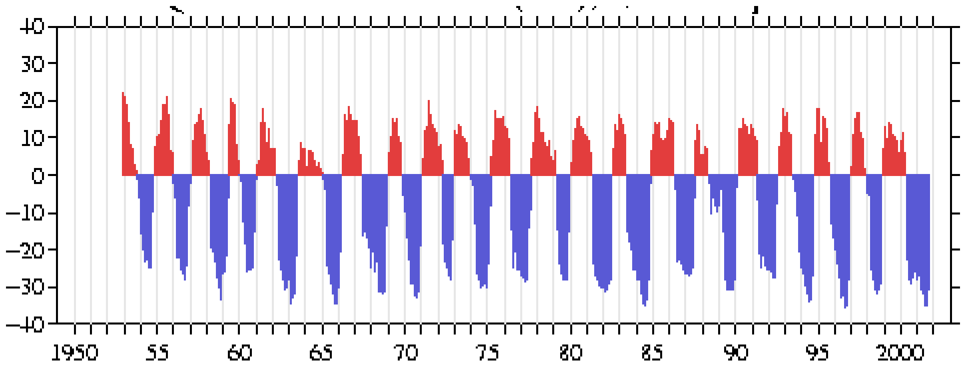


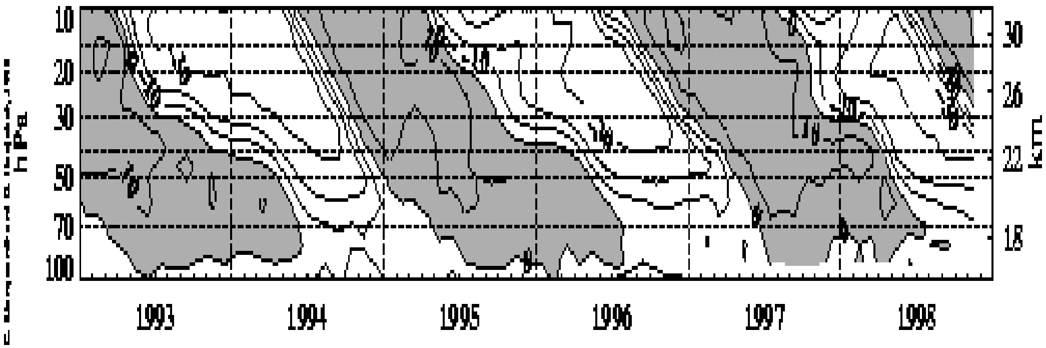
Figure 6. Time series of zonal-mean westerly winds over the equator, from November 1991 to February 1999. The tick marks along the x-axis mark each January, April, July and October. The additional lines show where the values are mainly derived from interpolated or climatogical data.

a) Observations: Equatorial waves and quasi biennal oscillation

Index de l'OQB
Moyenne du vent zonal (m/s) à 30hPa (~25km). Extrait du site:
tao.atmos.washington.edu



Moyenne zonale du vent zonale en fonction du temps et de l'altitude
Extrait du site:
tao.atmos.washington.edu



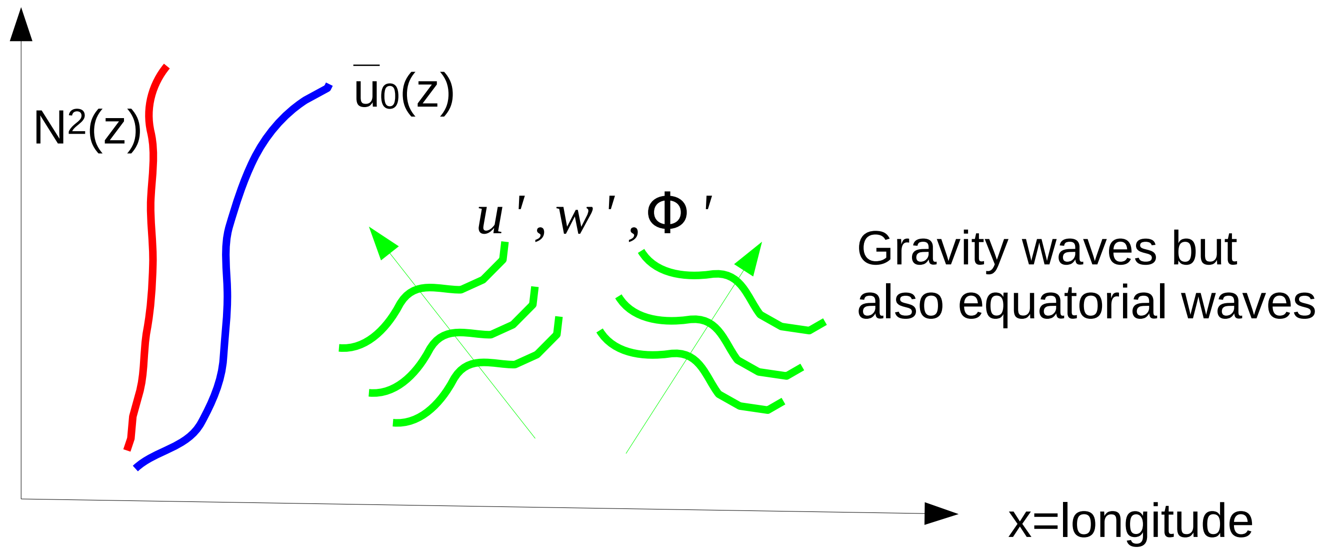
Noter la descente vers le bas des lignes de phase, indicatif que les modifications sont dues à des ondes venant d'en bas

b) Quasi-Biennal Oscillation and gravity waves mean flow interactions

Toy model 5: $N^2(z)\#0$, $u_0(z)\#0$, $f=0$, 2D(x-z)

Base equations are 2D, this will allows to handle variations in the vertical of the basic flow. Coriolis force is neglected : we are in the tropics

$z = H \ln(p_r/p)$, log pressure altitude



$$\frac{Du}{Dt} = -\frac{\partial \Phi_e}{\partial x} + X$$

$$\frac{\partial u}{\partial x} + \rho_0^{-1}\frac{\partial(\rho_0 w)}{\partial z} = 0$$

$$\frac{D}{Dt}\partial_z \Phi_e + N^2 w = Q$$

$$\Phi = \Phi_0(z) + \Phi_e(x, y, z, t)$$

$$N^2 = \Phi_{0zz} + \frac{\kappa}{H}\Phi_{0z}, \quad \rho_0(z) = \rho_s e^{-z/H}, \quad H = \frac{RT_s}{g} = 7\text{km}$$

$$\frac{D}{Dt} = \partial_t + u\,\partial_x + w\,\partial_z$$

b) Quasi-Biennal Oscillation and gravity waves mean flow interactions

Toy model 5: $N^2(z)\#0$, $u_0(z)\#0$, $f=0$, 2D(x-z)

Wave-mean flow separation

$$u(x,z,t)=\underbrace{\overline{u}_0(z)}_{O(1)}+\underbrace{u'(x,z,t)}_{O(\alpha)}+\underbrace{\overline{u}(z,t)-\overline{u_0}(z)}_{O(\alpha^2)}$$

$$\Phi(x,z,t)=\underbrace{\Phi_0(z)}_{O(1)}+\underbrace{\Phi'(x,z,t)}_{O(\alpha)}+\underbrace{\overline{\Phi}(z,t)-\Phi_0(z)}_{O(\alpha^2)}$$

Horizontal or zonal mean

$$\overline{u}=\frac{1}{L}\int_0^L u\,dx \quad \text{ou} \quad \overline{u}=\frac{1}{2\lambda}\int_0^{2\pi} u\,d\lambda$$

L size of the domain (périodic)

λ longitude

Equations for the wave (O(α)):

$$\left(\partial_t+\overline{u}_0\partial_x\right)u'+\overline{u}_{0z}w'=-\partial_x\Phi'+X'$$

$$\partial_x u'+\frac{1}{\rho_0}\partial_z\left(\rho_0 w'\right)=0$$

$$\left(\partial_t+\overline{u}_0\partial_x\right)\Phi'_z+N^2 w'=Q'$$

Equations for the mean flow (O(α^2)):

$$\partial_t\overline{u}=\frac{-1}{\rho_0}\partial_z\left(\rho_0\overline{u'w'}\right)+\overline{X}$$

$$\partial_t\overline{\Phi}_z=\frac{-1}{\rho_0}\partial_z\left(\rho_0\overline{\Phi'_z w'}\right)+\overline{Q}$$

$$\overline{w}=0$$

The fluxes $\overline{u'w'}$ et $\overline{\Phi'_z w'}$ translate the effect of the wave on the mean flow.

(consequence of the 2D geometry and from the continuity equation)

b) Quasi-Biennal Oscillation and gravity waves mean flow interactions

Toy model 5: $N^2(z)\#0$, $u_0(z)\#0$, $f=0$, 2D(x-z)

<u>Let us consider an adiabatic and inviscid wave</u>
(absence of breaking):

$$\left(\partial_t + \bar{u}_0 \partial_x\right) u' + \bar{u}_{0z} w' = -\partial_x \Phi'$$
$$\partial_x u' + \frac{1}{\rho_0} \partial_z\left(\rho_0 w'\right) = 0$$
$$\left(\partial_t + \bar{u}_0 \partial_x\right) \Phi'_z + \bar{N}^2 w' = 0$$

Let us analyse harmonics by harmonics and consider monochromatic waves in x and t
we allow the mean flow to vary in z

$$\begin{pmatrix} u' \\ w' \\ \Phi' \end{pmatrix} = \Re\left[\begin{pmatrix} \hat{u}(z) \\ \hat{w}(z) \\ \hat{\Phi}(z) \end{pmatrix} e^{i(kx-\omega t)}\right] e^{z/2H}$$

with k>0, and α<<1

Polarisation relations:

$$-i\hat{\omega}\hat{u} + \bar{u}_{0z}\hat{w} = -ik\hat{\Phi}$$
$$-i\hat{\omega}\left(\hat{\Phi}_z + \frac{\hat{\Phi}}{2H}\right) + N^2\hat{w} = 0$$
$$ik\hat{u} + \left(\hat{w}_z - \frac{\hat{w}}{2H}\right) = 0$$

Intrinsic frequency:

$$\hat{\omega} = \omega - k\bar{u}_0$$

Vertical structure Equation:
(Taylor-Goldstein)

$$\frac{d^2\hat{w}}{dz^2} + \underbrace{\left(\frac{\bar{N}^2 k^2}{\hat{\omega}^2} + \frac{k}{\hat{\omega}}\left(\bar{u}_{0zz} + \frac{\bar{u}_{0z}}{H}\right) - \frac{1}{4H^2}\right)}_{Q(z)}\hat{w} = 0$$

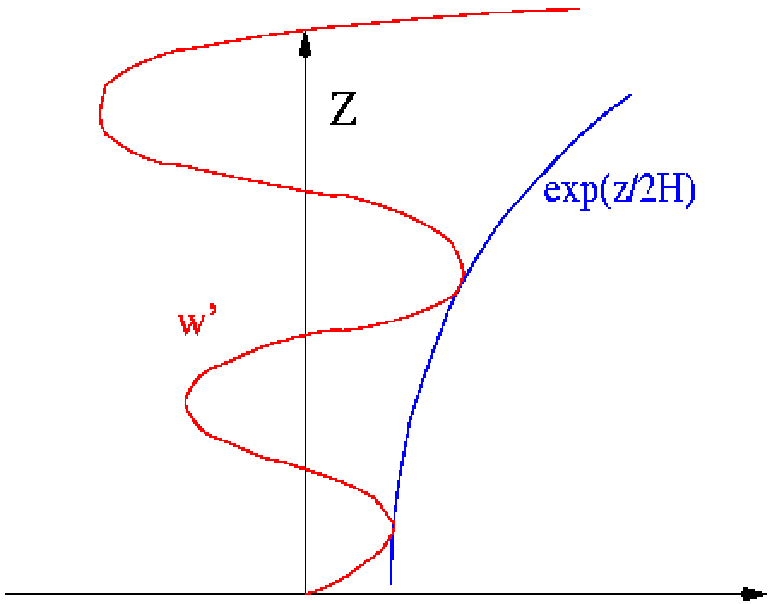
Q(z) is the Scorer parameter

b) Quasi-Biennal Oscillation and gravity waves mean flow interactions

Toy model 5: $N^2(z)\#0$, $u_0(z)\#0$, $f=0$, 2D(x-z)

The effect of density:

Structure of a wave when $\underline{u}_0$=cte, $\overline{N}^2$=cte $\omega\#0$.



$$\hat{w}(z)=\hat{w}(0)e^{imz}$$

$$w'=\Re\left[e^{i(kx+mz-\omega t)}\right]e^{z/2H}$$

$$m^2=\frac{\bar{N}^2k^2}{\hat{\omega}^2}-\frac{1}{4H^2}$$

where the intrinsic frequency $\hat{\omega}=\omega-k\bar{u}_0(z)$

Upward propagation :

$$\hat{C}_{gz}=\frac{\partial\hat{\omega}}{\partial m}=\frac{-m\hat{\omega}}{m^2+1/(4H^2)}>0 \rightarrow \quad m=-\text{sign}(\hat{\omega})\sqrt{\frac{\bar{N}^2k^2}{\hat{\omega}^2}-\frac{1}{4H^2}}$$

b) Quasi-Biennal Oscillation and gravity waves mean flow interactions

Toy model 5: $N^2(z)\#0$, $u_0(z)\#0$, $f=0$, 2D(x-z)

Thermal and momentum fluxes :

$$-\rho_0 \overline{w'\Phi'_z} = -\Re\left[\frac{\rho_s}{2}\hat{w}\left(\hat{\Phi}_z + \frac{\hat{\Phi}}{2H}\right)^*\right] = -i\,\Re\left[\frac{\rho_s}{2}\frac{N^2}{\hat{\omega}}\hat{w}\hat{w}^*\right] = 0$$

No heat flux:

$$\overline{F^z} = -\rho_O \overline{u'w'} = -\frac{\rho_s}{2}\Re\left[\hat{u}\hat{w}^*\right] = \Re\left[-i\frac{\rho_s}{2k}\hat{w}_z\hat{w}^*\right]$$

The waves can carry horizontal momentum vertically:

For instance when $\overline{u}_0$=cte, N^2=cte

$$\hat{w} = \hat{w}(0)e^{imz}$$

$$w' = \Re\left[e^{i(kx+mz-\omega t)}\right]e^{z/2h}$$

$$\overline{F^z} = \frac{\rho_s}{2}\frac{m}{k}\hat{w}(0)\hat{w}(0)^* \neq 0$$

It has a sign opposite to the waves horizontal phase speed

$$\hat{C}_x = \frac{\hat{\omega}k}{|\vec{k}|^2}$$

b) Quasi-Biennal Oscillation and gravity waves mean flow interactions

Toy model 5: $N^2(z)\#0$, $u_0(z)\#0$, $f=0$, 2D(x-z)

The non-interaction Eliassen Palm theorem

From the expression of the momentum flux: $$\overline{F^Z} = \Re\left[-i\frac{\rho_s}{2k}\hat{w}_z\hat{w}^*\right]$$

And the Taylor Goldstein equation: $\hat{w}_{zz} + Q\hat{w} = 0$ multiplied by $\hat{w}^*$

One deduce that: $$\frac{\partial \overline{F}^z}{\partial z} = 0$$

A linear stationnary adiabatic and non-dissipated wave do not modify the mean flow

Remember that $$\partial_t \overline{u} = \frac{-1}{\rho_0}\partial_z\left(\rho_0 \overline{u'w'}\right) + \overline{X} = \frac{1}{\rho_0}\frac{\partial \overline{F}^z}{\partial z} + \overline{X}$$

b) Quasi-Biennal Oscillation and gravity waves mean flow interactions

Toy model 5: $N^2(z)\#0$, $u_0(z)\#0$, $f=0$, 2D(x-z)

Value and sign of the EP-flux

$$m(z) = -sign(\hat{\omega})\sqrt{Q(z)} \longrightarrow \boxed{sign(\bar{F}^z) = -sign(k\hat{\omega})}$$

The chosen sign for m is to guaranty Upward propagation

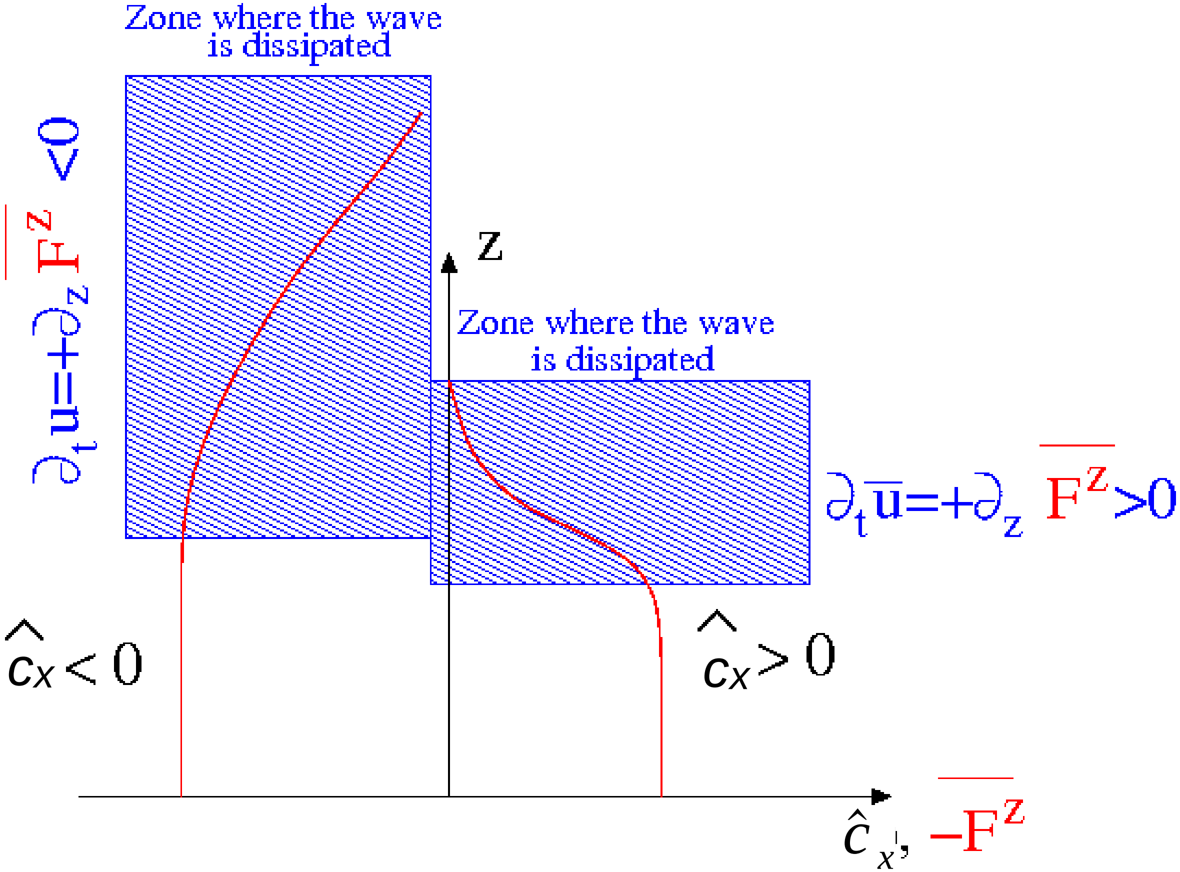
This imposes as well the sign of the EP flux

Eastward propagating waves accelerate the flow when they break

$$\hat{c}_x = \hat{\omega}\, k/|\vec{k}|^2 > 0$$

Westward propagating waves accelerate the flow when they break

$$\hat{c}_x = \hat{\omega}\, k/|\vec{k}|^2 < 0$$



b) Quasi-Biennal Oscillation and gravity waves mean flow interactions

Toy model 5: $N^2(z)\#0$, $u_0(z)\#0$, $f=0$, 2D(x-z)

Solution WKB: Breaking

$$\left|\hat{\Phi}_{zz}+\frac{\kappa}{H}\hat{\Phi}_z\right|e^{z/2H}>N^2$$

By using the polarization relationships:

$$|\hat{w}|<\left|\frac{\hat{\omega}}{m}\right|e^{-z/2H}=w_s(z)$$

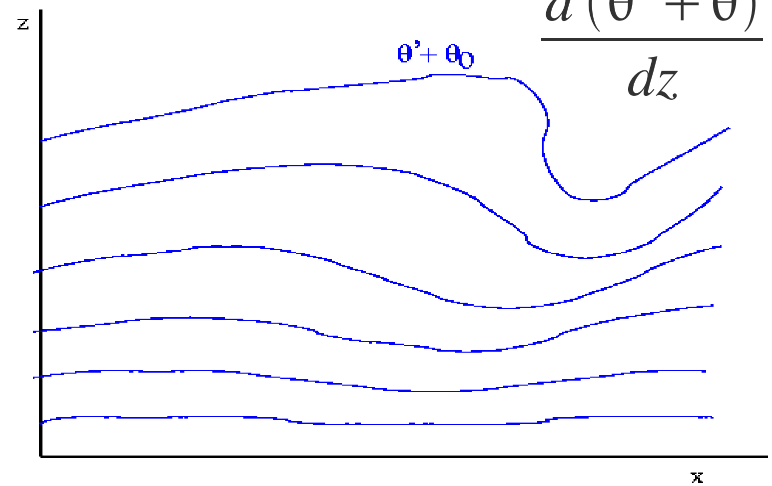
Or if you put $w_S(z)$ into the EP-flux expression:

$$\left|\overline{F^z}\right|<\left|\overline{F_S^z}\right| \quad \text{où} \quad \overline{F_S^z}=-\frac{\rho_r\hat{\omega}^3}{2k^2N}e^{-z/H}$$

There is no flux through critical levels (e.g. levels where $\hat{\omega}=0$)

For a constant flow the breaking altitude is:

$$Zbr=2H\ln\left(\frac{\hat{\omega}^2}{Nk|\hat{w}(0)|}\right)$$



$$\frac{d(\theta'+\bar{\theta})}{dz}<0$$

- Breaking is favored when the amplitude of the intrinsic phase speed is small:

  - In westward flows (U>0), breaking of westward waves (C>0) is favored. They accelerate the flow when they break.

  - In an eastward flow (U<0), breaking of eastward waves is favoured. They decelerate the flow then.

b) Quasi-Biennal Oscillation and gravity waves mean flow interactions

Numerical solution

$$\rho_0 \frac{\partial \overline{u}}{\partial t} = \sum_{i=1}^{2} \frac{\partial \overline{F}_i^z}{\partial z} + \nu \frac{\partial^2 \overline{u}}{\partial z^2}$$

Just two waves: one eastward and one westward

1)We impose in z=0 (here the tropopause): $\overline{F_i^z}(0), \omega_i, k_i$

2) Passage from z to z+dz

z+dz

z

Vertical levels

$$\overline{F}_i^z(z+dz) = \overline{F}_i^z(z)$$

$$si \left|\overline{F}_i^z(z+dz)\right| > \left|\overline{F}_{iS}^z(z+dz)\right|$$

$$alors: \overline{F}_i^z(z+dz) = \overline{F}_{iS}^z(z+dz)$$

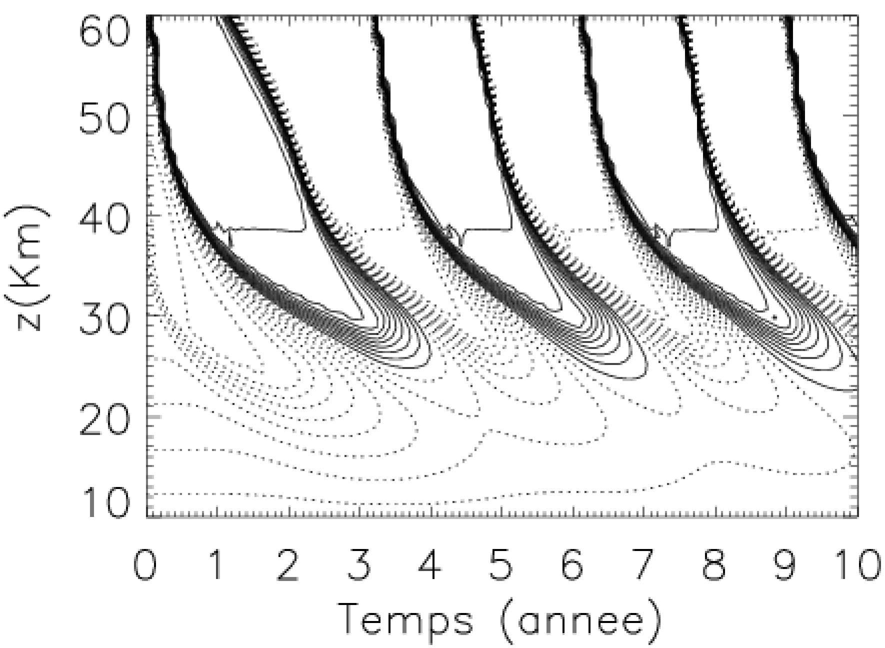
3) Evaluation of $\overline{u}(t+dt, z)$

4) $\overline{u}(t+dt, z) \rightarrow \overline{u}_0(z)$

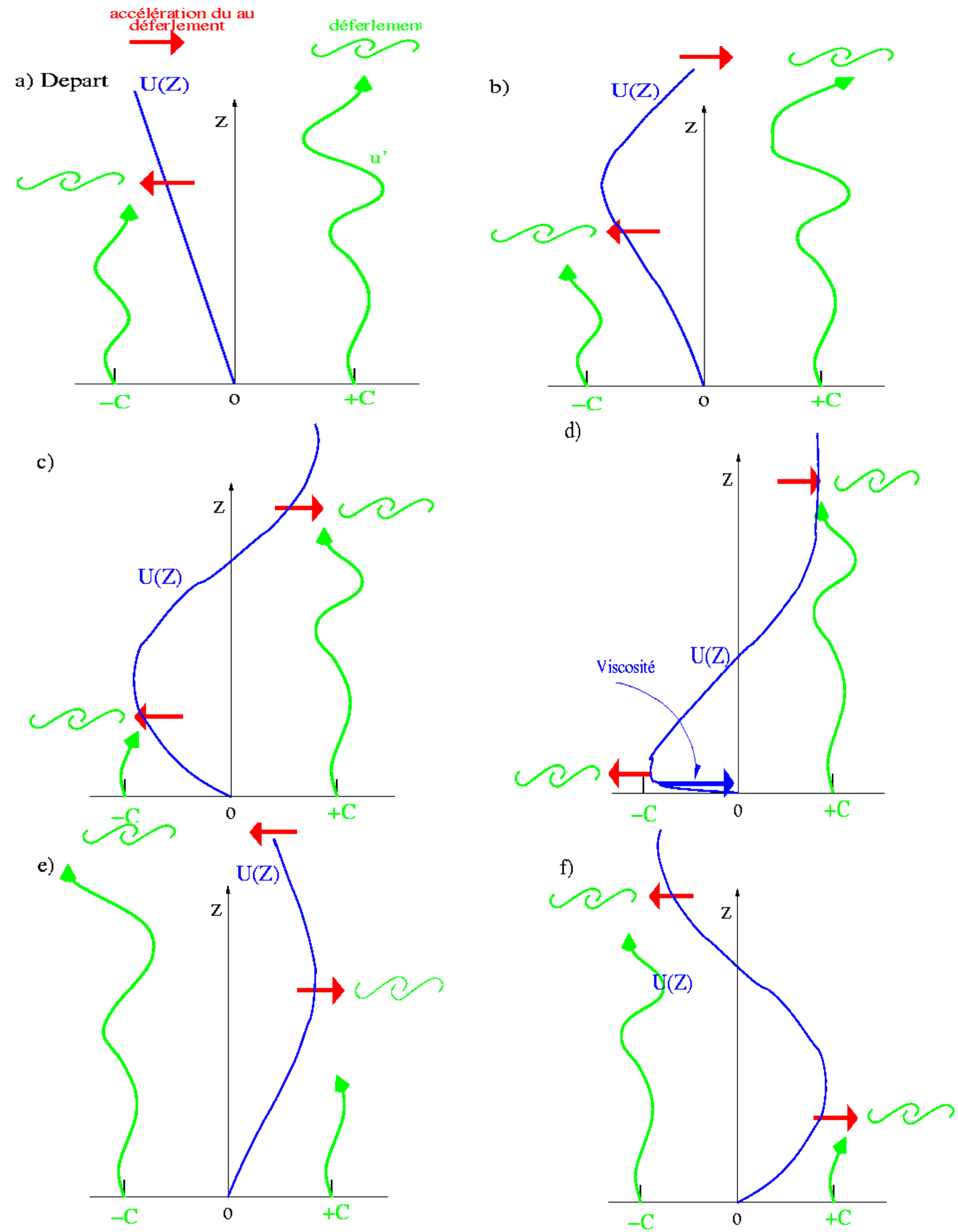
b) Quasi-Biennal Oscillation and gravity waves mean flow interactions

Résult from the model:

$\bar{u}(z,t)$



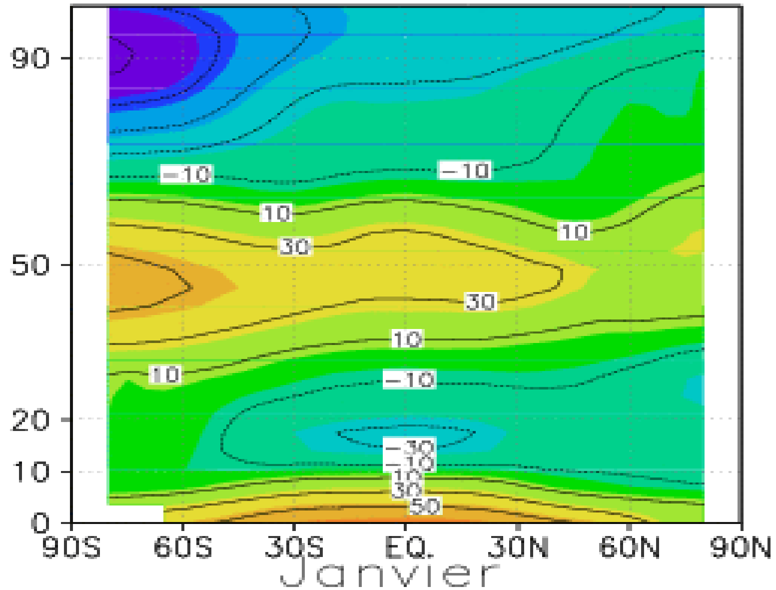
Interpretation:



C) Returning to the Temperature minimum at the summer polar mesopause

The middle atmosphere is not in radiative equilibrium

January Temperature (CIRA)



At the mesopause in NH winter the eastward phase speed waves accelerate the zonal mean flow in the SH
The westward phase speed waves decelerate the flow in the NH

- Temperature is far from the radiative equilibrium:
- During solstices:
  - In the upper mesosphere (70-90km) T increases from the winter pole to the summer pole !!!
  - At the mesopause (90km) over the summer pole is the coldest region of the neutral atmosphere!