Dynamic Meteorology

(WAPE: General Circulation of the Atmosphere and Synoptic Meteorology)

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1) Zonal mean climatologies and fundamental equations

a)Thermodynamical and chemical vertical profiles

Static relations

b)Zonal mean climatology

Zonal wind and T, Thermal wind balance

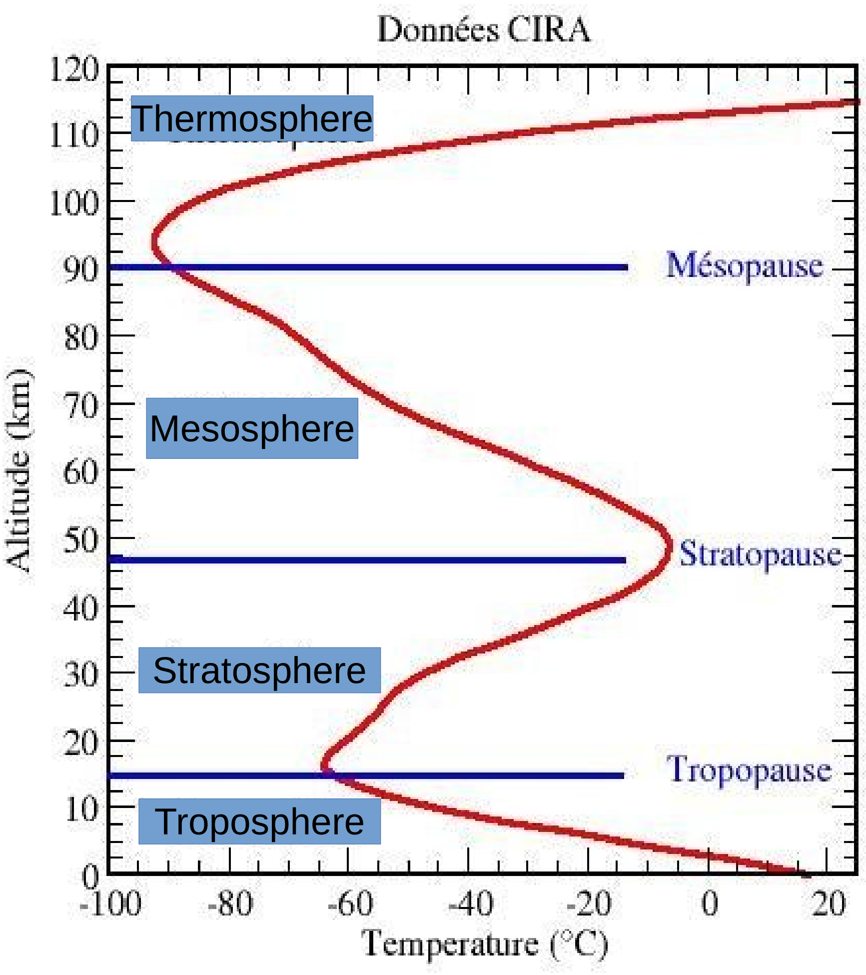
c)Equations of motion

Primitive Eqs on the sphere: hydrostatic in log-P coordinate, Boussinesq

a) Thermodynamical and chemical vertical profiles

Vertical structure of the atmosphere:

Temperature as a function of the geometrical altitude z*

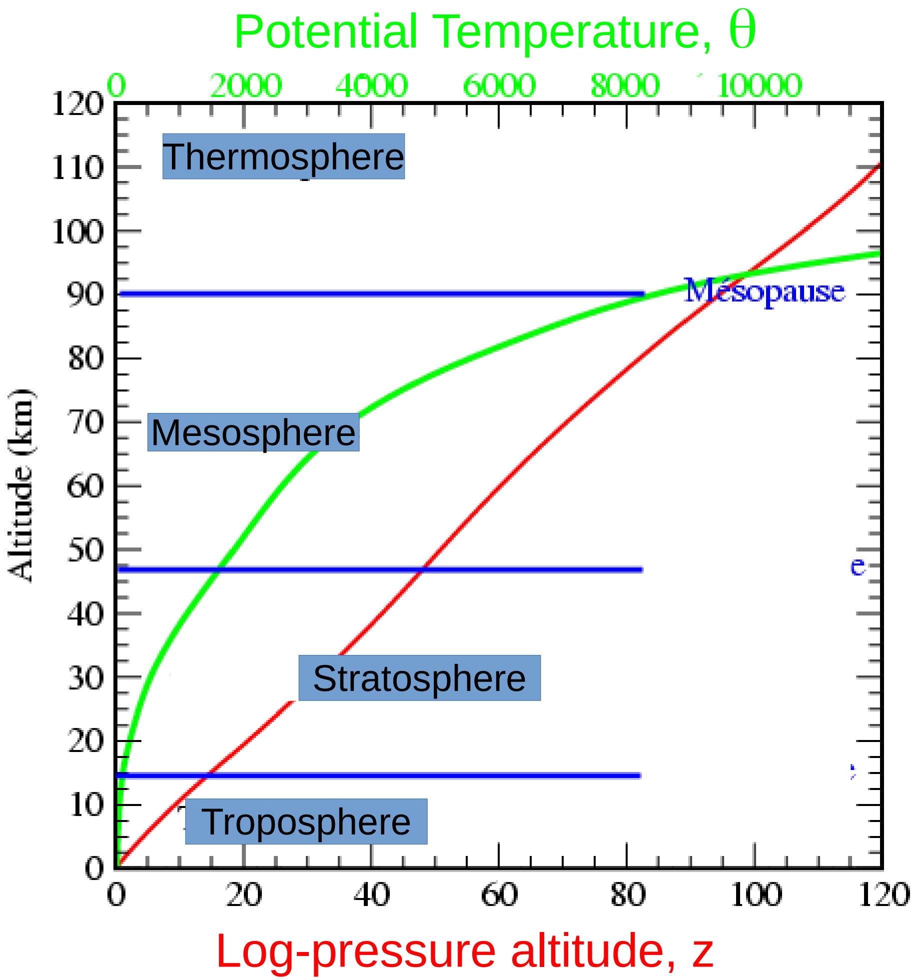


- CIRA data (1988): Rockets, rasiosondes, satellites above 110km.
- Above 90km lies the thermosphere, a layer highly exposed to sun radiation and the X-rays ionised the constituents. It contains the ionosphere (80-500km) where Aurora Borealis occur.
- Extremely thin in mass, T can vary from 600K to 1800K in one day (solar tides)
- The atmosphere there is not neutral and is not well mixed: the composition varies according to the mass of the molecules because of the large distances between them.
- The middle atmosphere:
  - contains 2 of the 3 layers of the neutral atmosphere: the stratosphere and the mesosphere.
  - Les chemical major constituant are still well mixed
  - There is a Max in T at 50km due to Ozone (0_3). This defines the stratopause separating the stratosphere and the mesosphere
- Troposphere: T decreases with z*, the heating comes from the surface, whereas the wator vapor and clouds cool the atmosphere above via Infrared Radiation (IR).

a) Thermodynamical and chemical vertical profiles

Static relations: P, ρ, θ et Φ as a function of T(z*)

CIRA Data



- Perfect gas law + hydrostatic relation:

$$\frac{p}{\rho} = RT \quad \text{et} \quad \frac{\partial p}{\partial z^*} = -\rho g$$

- Pressure and Temperature:

$$p = p_s \exp\left(-\int_{z_s^*}^{z^*} \frac{g}{RT} dz^*\right)$$

- Log-pressure altitude:

$$z = H \ln\left(\frac{p_s}{p}\right)$$

- Characteristic height: $H = \frac{R T_s}{g} = 7\text{km}$

- *T_s*~240K,(*mean characteristic Temperature)*
  z=z* si *T=T_s*

- Potential Temperature: $\theta = T\left(\frac{p_s}{p}\right)^{\kappa} = T\exp\left(\frac{\kappa z}{H}\right)$

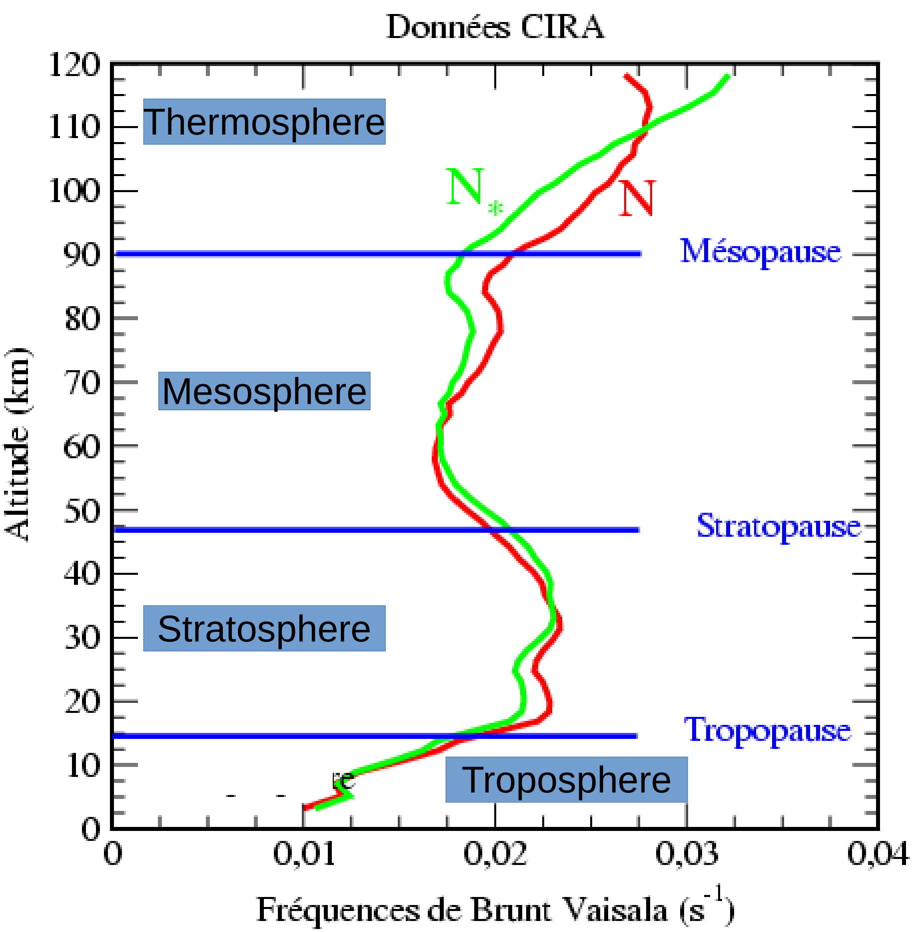
- Geopotential: $\Phi = \int_{z_s^*}^{z^*} g dz^*$

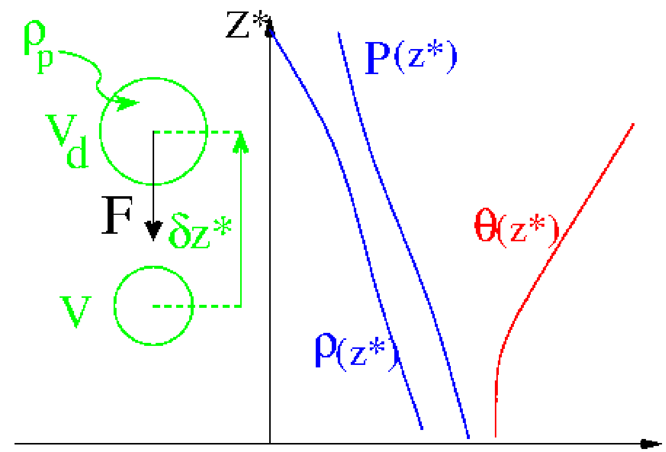
- Hydrostatic relation: $\frac{\partial \Phi}{\partial z} = \frac{RT}{H}$

a) Thermodynamical and chemical vertical profiles

Static stability:

Parcel method: we displace adiabatically a volume V of air and consider that the pressure P adjusts to the environment (***P_p***=*$P(z^*+\delta z^*)$*)!!





$$F=-g\left(\rho(z^*)V-\rho(z^*+\delta z^*)V_d\right)=-gm\left(1-\frac{\rho(z^*+\delta z^*)}{\rho_p}\right)$$

$$=-gm\left(1-\frac{T_p}{T(z^*+\delta z^*)}\right)=-gm\left(1-\frac{\theta(z^*)}{\theta(z^*+\delta z^*)}\right)$$

Vertical acceleration: $$\frac{d^2\delta z^*}{dt^2}=\frac{F}{m}=-\frac{g}{\theta}\frac{d\theta}{dz^*}\delta z^*$$

Brunt Vaisala Frequency: $$N_*^2=\frac{g}{\theta}\frac{d\theta}{dz^*}$$

Major trace species (1): CO_2

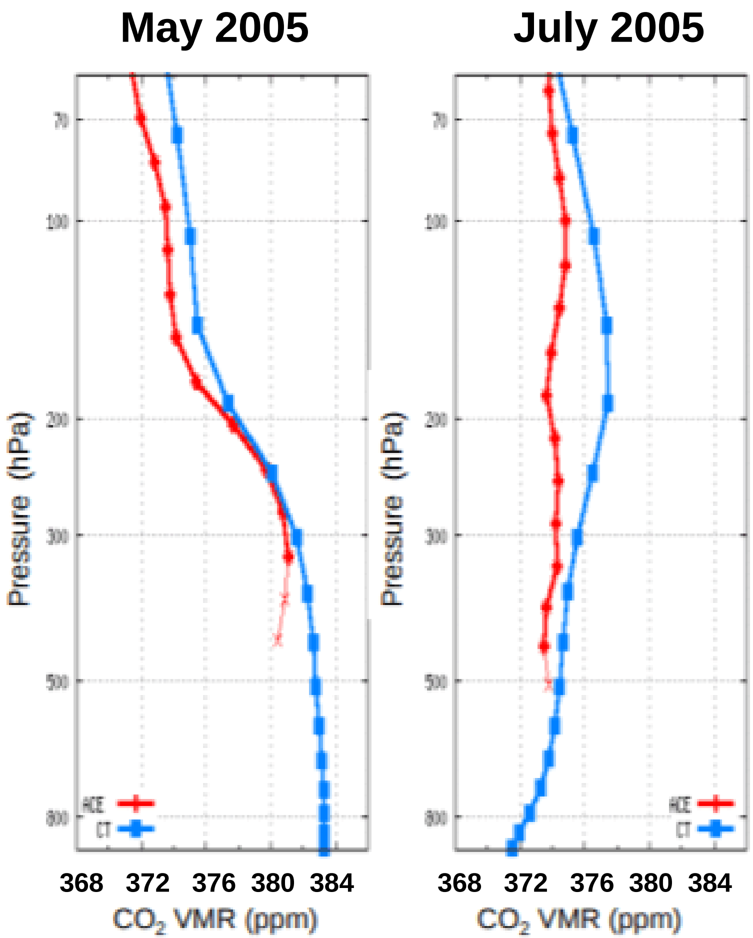


Fig. 7. CO_2 concentration (x-axis in ppm) evolution with pressure (y-axis in hPa) for May (left) and July (right) for 2005 for the 50° N–60° N latitude band. ACE-retrieved profile is in red and CT profile is in blue. The thin red line indicates that less than 10 occultations were available. ACE CO_2 standard deviation at each altitude (not shown in the picture) fluctuates around 3 ppm (±1 ppm).

CO_2 (carbon dioxide):

- Satellite observations
- Almost uniformly mixed and very active in the infrared
- Today global measurements permit to measure small variations related to sources and sinks (Urban areas, forest, oceans, ect…)
- Very small seasonal variations due to vegetation cycle (here between may and july)

$$\nu_{CO_2} = \frac{P_{CO_2}}{p}$$

a) Thermodynamical and chemical vertical profiles

Major trace species (2): H_2O

Satellite measurements in the stratosphere

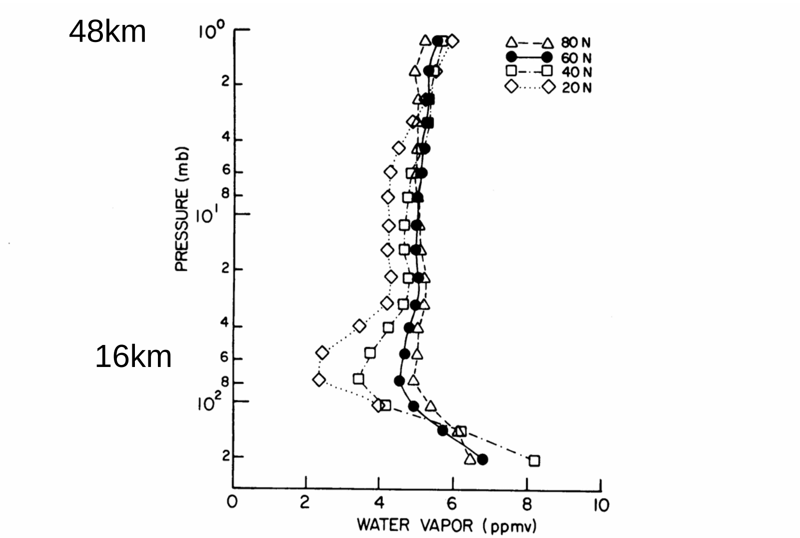
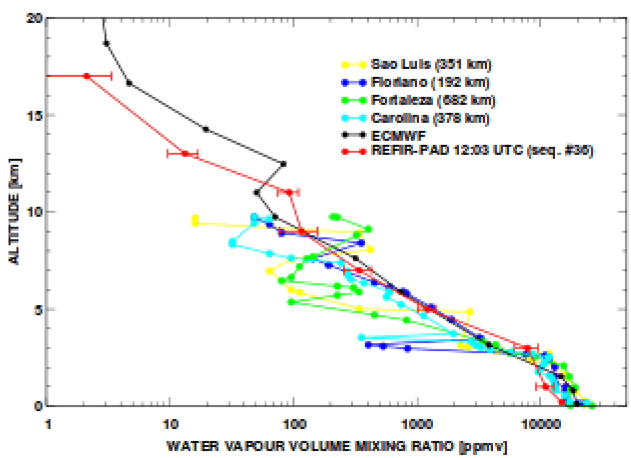


Fig. 1.5. Vertical profiles of water vapor mixing ratio at several latitudes measured by the LIMS instrument on the *Nimbus* 7 satellite for May 1–26, 1979. [From Remsberg *et al.* (1984b). American Meteorological Society.]

Balloon measurements in the troposphere:

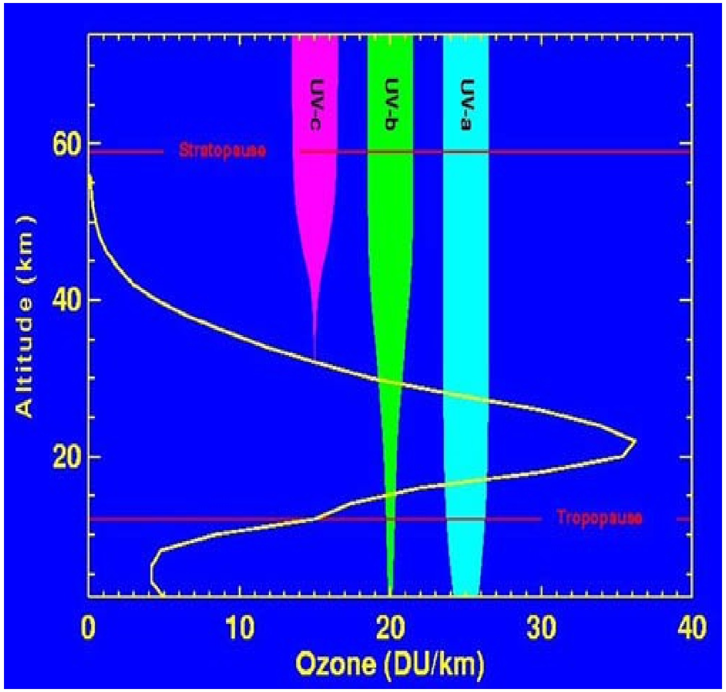


H2O (Wator vapour):

- Rapid decay with altitude, very weak values (almost uniform) in the stratosphere.
- Strong greenhouse impact in the troposphere.
- Note the minimum value at the equatorial tropopause

$$\nu_{H_2 0} = \frac{p_{H_2 O}}{p}$$

Major trace species (3), Ozone: O_3



Ozone profile in the midlatitudes and penetration of the UV-a, UV-b, UV-c

- Absorption of Uv-b's by 0_3 is the main driver of the middle atmosphere (stratosphere + mesosphere) circulation.
- Ozone protects us from the UV-b
- Maximum at 30-40km, in the stratosphere.

a) Thermodynamical and chemical vertical profiles

The Ozone heating makes up the stratosphere above the tropopause.

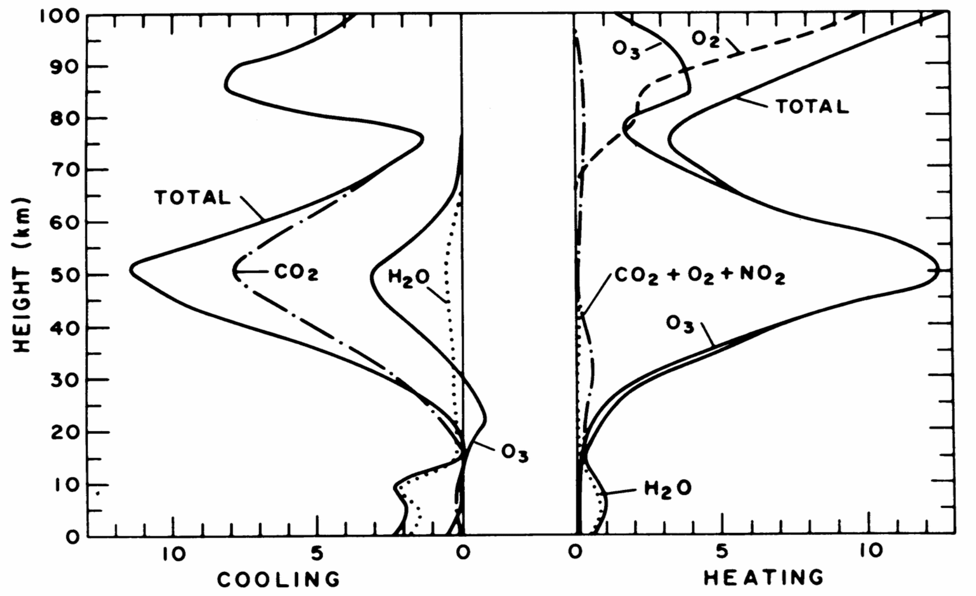
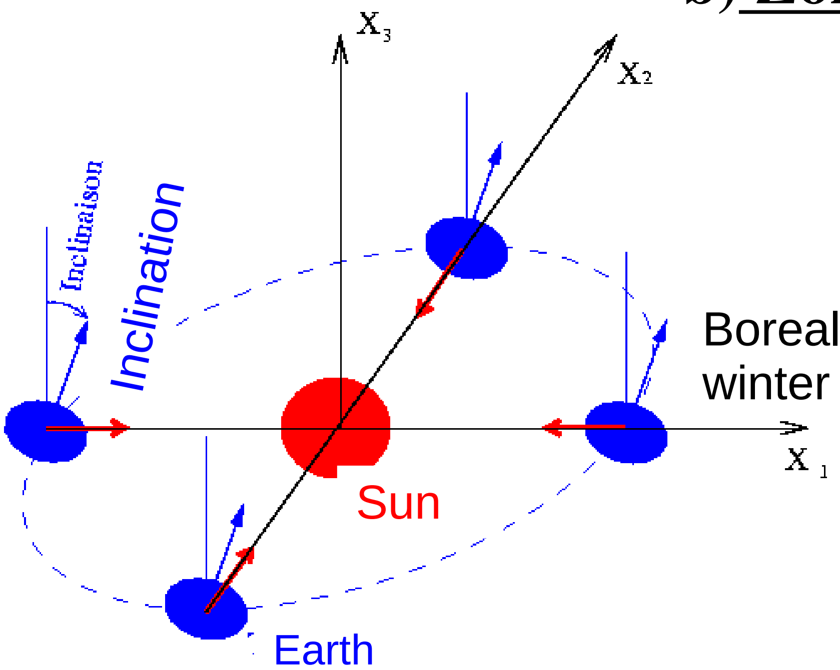


Fig. 2.1. Vertical distribution of heating due to absorption of solar radiation (right) and cooling due to emission of infrared radiation (left). [From London (1980), with permission.]

Vertical distribution of the heating due to absorption of the solar radiation and of the cooling due to emission of Infrared radiation (major thermal forcings in the middle atmosphere)

Thermal forcing in the troposphere are forced by the surface, combine infrared absorption (greenhouse effect), and convection

b) Zonal mean climatology

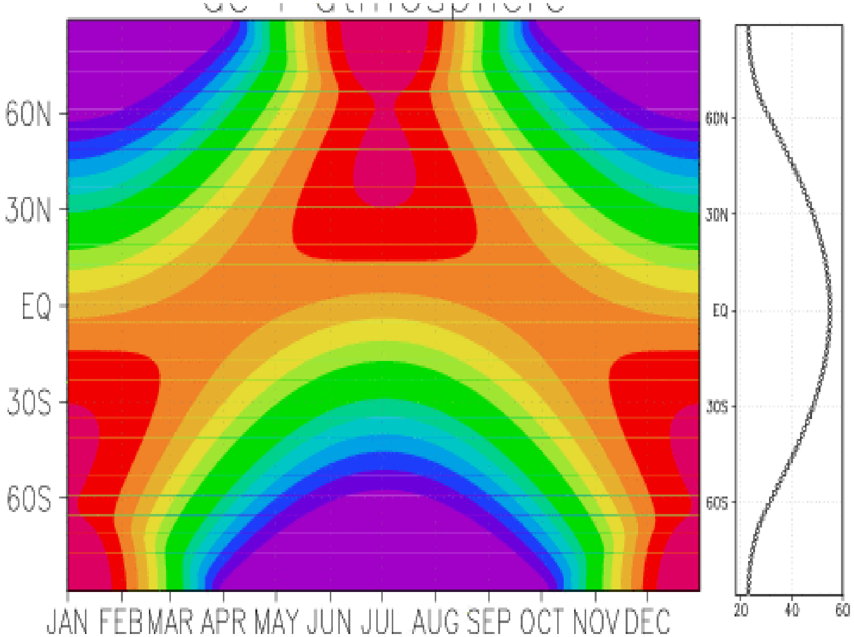


Seasonal cycle of the solar flux

- O_3 re-emit almost instantaneously producing a chemical heating, the UV it absorbs
- The solar flux is maximum at the poles in summer, in part because the length of the day is 24h there
- Averaged over the year, the solar flux is maximum at the equator

Incoming solar flux

Annual mean

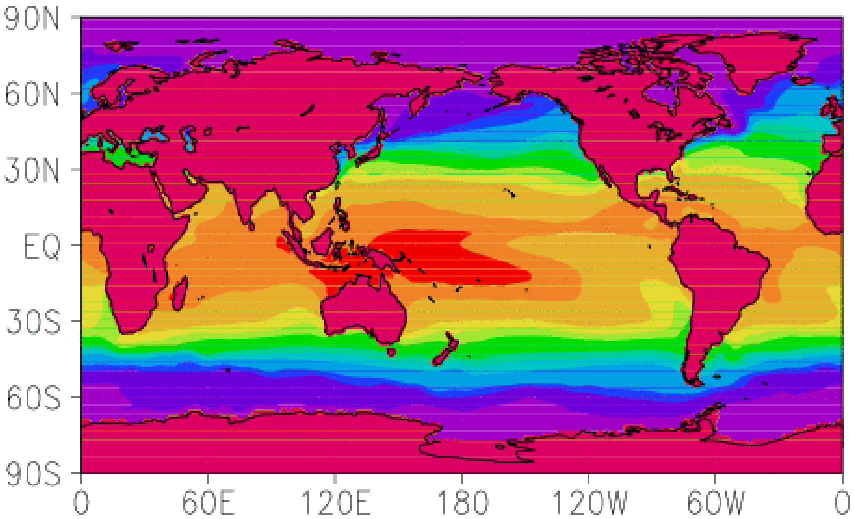


b) Zonal mean climatology

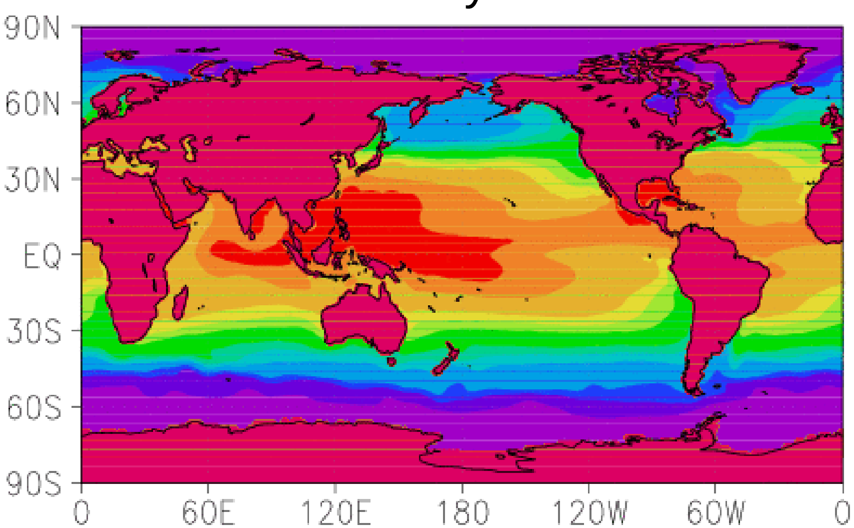
The heat capacity of the ocean is very large, it allows the oceans to integrate the solar cycle. The thermal forcing is then in the IR, and absorbed by H_2O and CO_2 before reaching the middle atmosphere (greenhouse effect)

Sea Surface Temperature (ECMWF 1993-1997)

January



July



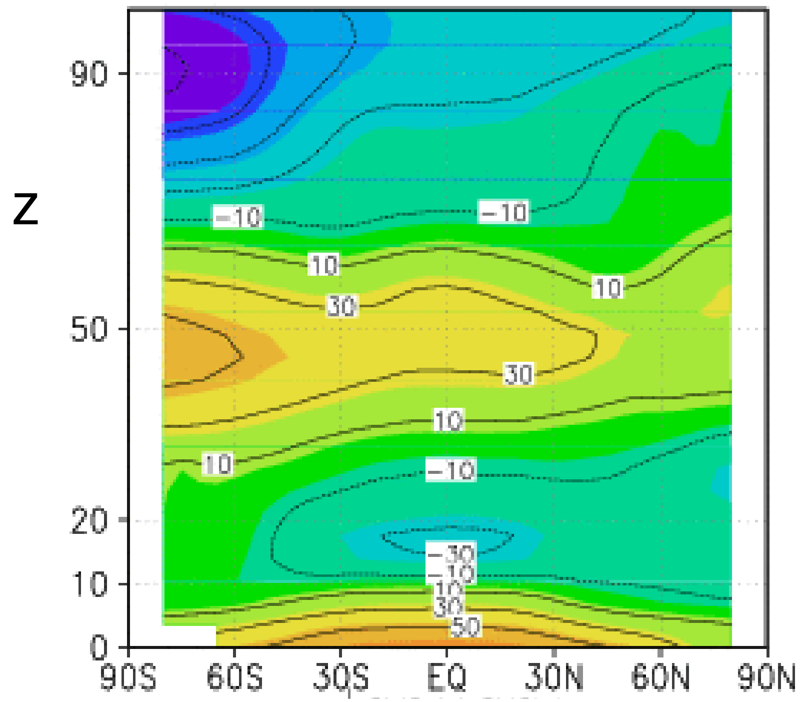
- The SST is always warmer in the equatorial regions
- It maintains a large humidity in the tropical regions, yielding a large greenhouse effect there.
- The troposphere is essentially forced from below, and will experience a less dramatic annual cycle than the middle atmosphere

b) Zonal mean climatology

$$\bar{T} = \frac{1}{2\pi}\int_0^{2\pi} T\, d\lambda$$ Temperature (CIRA dataset)

$\bar{T}$(K)-230

January

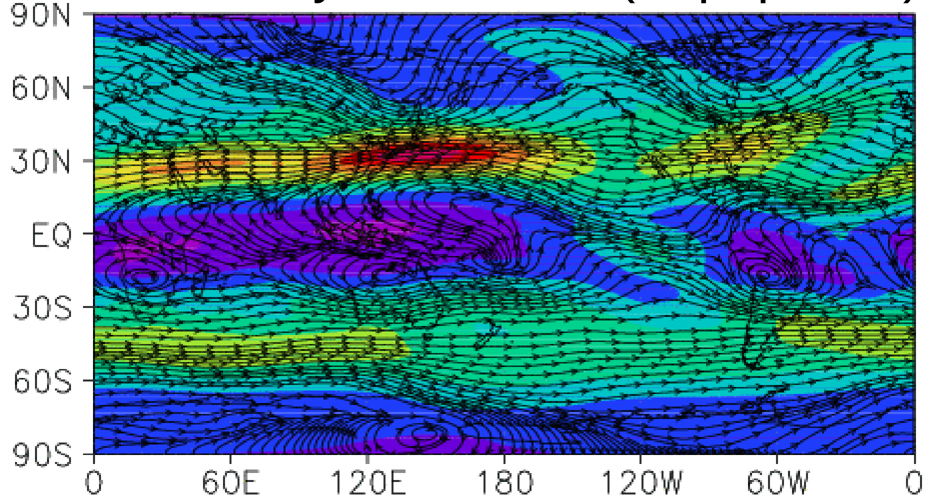


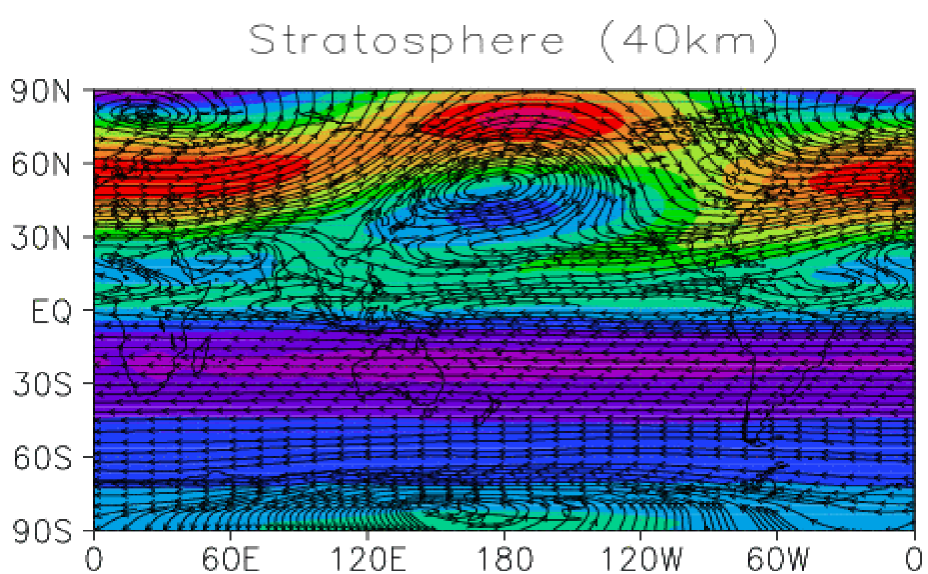
- Temperature decays with altitude in the troposphere.
- There is a minimum at the tropical tropopause (a greenhouse effect due to the presence of water vapour).
- In the stratosphere (20km<z<50km), T decreases from the summer pole to the winter pole.
- At the stratopause (50km) in the summer hemisphere, there is a max in T.
- Upper mesosphere (70-90km): T increases from the summer pole to the winter pole!
- A pronounced minima in T (~180K) over the summer pole!!

b) Zonal mean climatology

A striking illustration of the differences in circulation between the troposphere and the stratosphere

ECMWF (93-97) horizontal wind
January at z=12km (tropopause)





Winter time mean climatologies:

- The winds in the troposphere are eastward in both hemisphere and in the mid-latitudes.
- In the stratosphere the winds are eastward in the winter hemisphere and westward in the summer hemisphere.

b) Zonal mean climatology

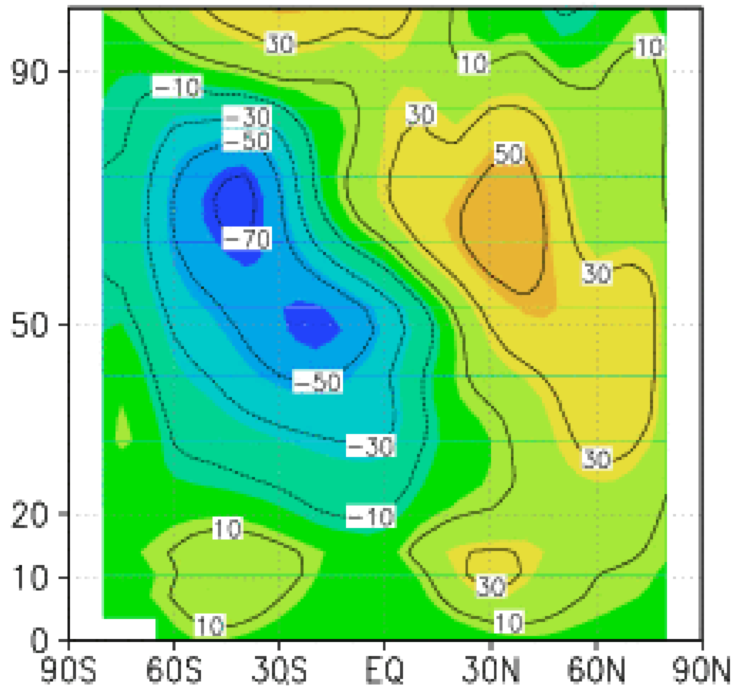
$$\bar{u} = \frac{1}{2\pi} \int_0^{2\pi} u \, d\lambda$$

Zonal wind (CIRA dataset)

$\bar{u}$(m/s)

January

z



Latitude (ϕ)

- Two westerly jets near below the subtropical tropopause. These westerlies extent almost down to the surface (0-16km) and characterize the midlatitude circulations in the troposphere.
- Still in troposphere, the winds tend to be slightly westward (easterly) in the tropics (trade winds).
- In the middle atmosphere (20-90km), the winds are eastward (westerlies) in the winter hemisphere and westward in the summer hemisphere.

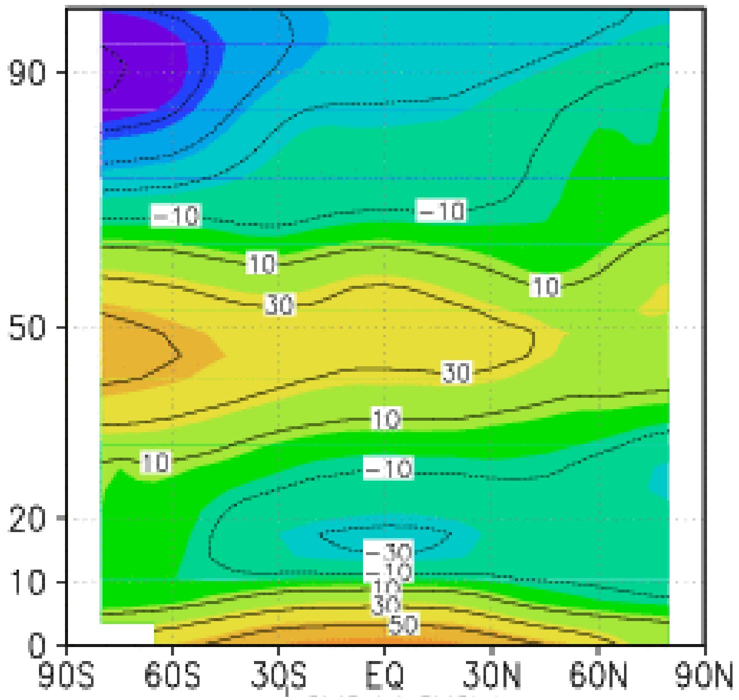
b) Zonal mean climatology

A first hint that thermal effect produces the winds : the thermal wind balance

$$2\Omega\sin(\phi)\frac{\partial \bar{u}}{\partial z}=-\frac{R}{aH}\frac{\partial \bar{T}}{\partial \phi}$$

$\bar{T}$(K)-230
January

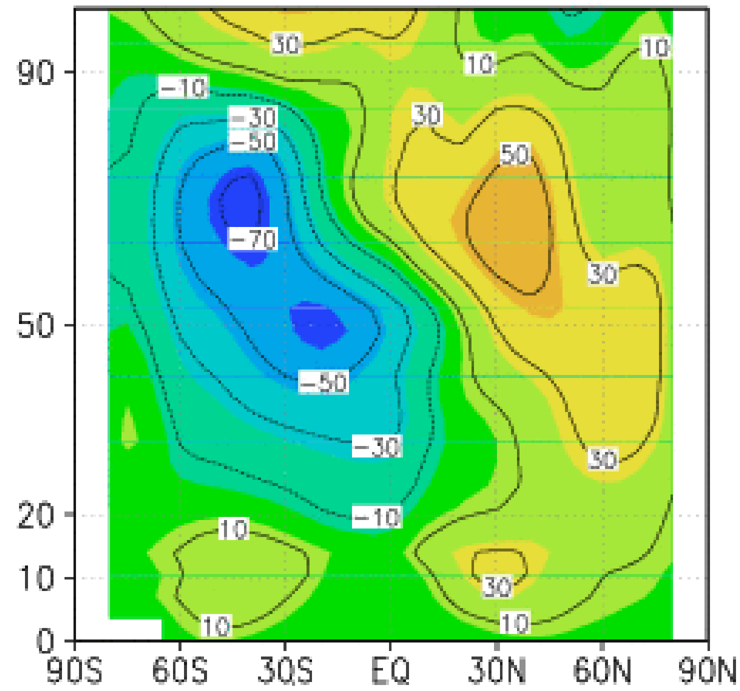
z



Latitude (ϕ)

$\bar{u}$(m/s)
January

z



Latitude (ϕ)

c) Equations of motion

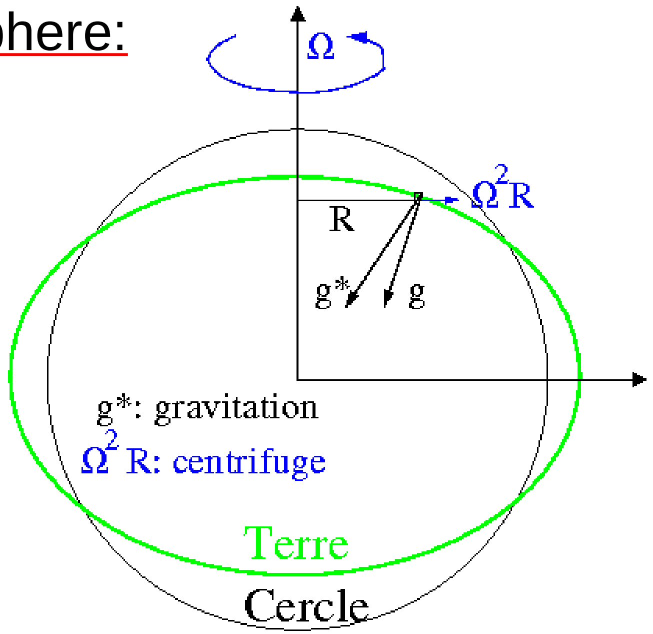
Newton law applied to hydrodynamic over a rotating sphere:

$$\frac{D\vec{U}}{Dt} + 2\vec{\Omega} \times \vec{U} = -\frac{1}{\rho}\vec{\nabla}p + \vec{g} + \vec{D}$$

$\vec{D}$: Frictional forces

p: pressure

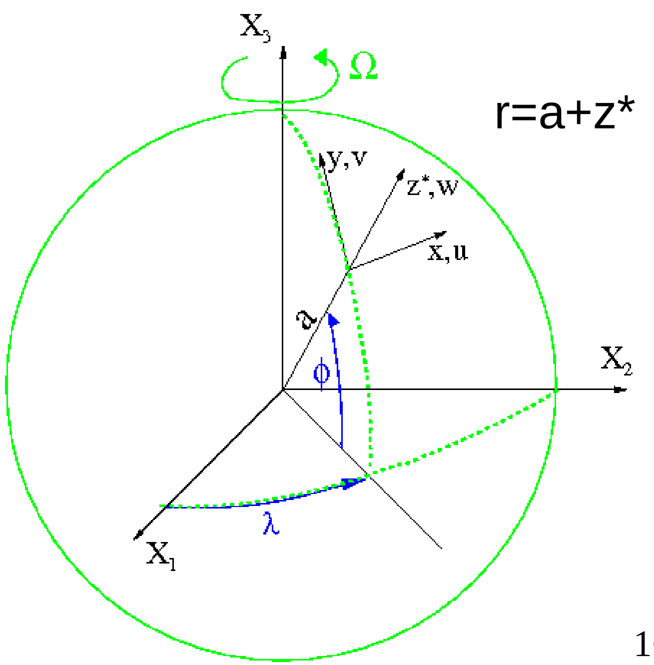
$\vec{g}$: Gravitation with a correction to include the centrifugal acceleration



Acceleration in spherical coordinates:

$$\frac{D\vec{U}}{Dt} + 2\vec{\Omega} \times \vec{U} = \begin{cases} \frac{Du}{Dt} - \frac{uv\tan\phi}{r} + \frac{u\mathrm{w}}{r} \\ \quad -2\Omega\sin\phi v + 2\Omega\cos\phi \mathrm{w} \\ \frac{Dv}{Dt} + \frac{u^2\tan\phi}{r} + \frac{v\mathrm{w}}{r} \\ \quad +2\Omega\sin\phi u \\ \frac{D\mathrm{w}}{Dt} - \frac{u^2+v^2}{r} - 2\Omega\cos\phi u \end{cases}$$

r=a+z*



Material derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{u}{r\cos\phi}\frac{\partial}{\partial\lambda} + \frac{v}{r}\frac{\partial}{\partial\phi} + \mathrm{w}\frac{\partial}{\partial r}$$

Wind components:

$$u = r\cos\phi\frac{D\lambda}{Dt},\ v = r\frac{D\phi}{Dt},\ \mathrm{w} = \frac{Dr}{Dt}.$$

c) Equations of motion

Approximate form for "large scale" motions a thin atmosphere (z*<<a, w<<*u*,*v*)

$$\frac{D\vec{U}}{Dt} + 2\vec{\Omega} \times \vec{U} \approx \begin{cases} \frac{Du}{Dt} - 2\Omega \sin\phi v - \frac{uv\tan\phi}{a} \\ \frac{Dv}{Dt} + 2\Omega \sin\phi u + \frac{uu\tan\phi}{a} \\ 0 \end{cases}$$

Thin : *r-> a*

Hydrostatic (w<<u,v)

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial}{\partial\lambda} + \frac{v}{a}\frac{\partial}{\partial\phi} + \mathrm{w}\frac{\partial}{\partial z^*}$$

$$: \; u = a\cos\phi\frac{D\lambda}{Dt}, \;\; v = a\frac{D\phi}{Dt}, \;\; \mathrm{w} = \frac{Dz^*}{Dt}.$$

Still in this approximation, the vertical component of the Newtown law reduce to the hydrostatic balance:

$$\frac{\partial p}{\partial z^*} = -\rho g$$

We can therefore use the Log pressure altitude as a vertical coordinate:

$$z = H\ln\left(\frac{p_s}{p}\right)$$

Material derivatives become:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{u}{a\cos\phi}\frac{\partial}{\partial\lambda} + \frac{v}{a}\frac{\partial}{\partial\phi} + w\frac{\partial}{\partial z}$$

With "vertical velocity" :

$$w = \frac{Dz}{Dt}$$

c) Equations of motion

Pressure force expressed using the log-altitude z:

$$\delta p \approx \left(\frac{\partial p}{\partial \lambda}\right)_{\phi, z^*} \delta\lambda + \left(\frac{\partial p}{\partial \phi}\right)_{\lambda, z^*} \delta\phi + \left(\frac{\partial p}{\partial z^*}\right)_{\lambda, \phi} \delta z^*$$

For a very small variation on a surface with λ=cte and for a surface z=cte, δp=0:

$$\delta p = 0 \approx \left(\frac{\partial p}{\partial \lambda}\right)_{\phi, z} \delta\phi - \frac{1}{\rho}\delta\Phi \longrightarrow \left(\frac{\partial \Phi}{\partial \lambda}\right)_{\varphi, z} = \lim_{\delta\varphi \to 0}\left(\frac{\delta \Phi}{\delta \lambda}\right)_{\varphi, z} = \frac{1}{\rho}\left(\frac{\partial p}{\partial \lambda}\right)_{\varphi, z^*}$$

Horizontal pressure gradient in polar coordinates :

$$-\frac{1}{\rho}\begin{pmatrix} \frac{1}{a\cos\phi}\left(\frac{\partial p}{\partial \lambda}\right)_{\phi, z^*} \\ \frac{1}{a}\left(\frac{\partial p}{\partial \phi}\right)_{z^*, \lambda} \end{pmatrix} = -\begin{pmatrix} \frac{1}{a\cos\phi}\left(\frac{\partial \Phi}{\partial \lambda}\right)_{\phi, z} \\ \frac{1}{a}\left(\frac{\partial \Phi}{\partial \phi}\right)_{z, \lambda} \end{pmatrix}$$

Mass conservation:

$$\delta M = \rho\delta x\delta y\delta z^* = \rho_0\delta x\delta y\delta z \text{ where } \rho_0 = \rho_s e^{-z/H}$$

$$\frac{1}{\delta M}\frac{D\delta M}{Dt} = \frac{1}{\rho_0}\frac{D\rho_0}{Dt} + \left(\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z}\right) = 0 \longrightarrow \mathrm{div}\rho_0\vec{u} = 0$$

Thermodynamics (approximation: heating applied to “perfect” dry):

$$\frac{DQ}{Dt} = C_p\frac{DT}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt} = \dot{Q} \longrightarrow \frac{DT}{Dt} + W\frac{\kappa T}{H} = \frac{\dot{Q}}{C_p} \quad \text{or} \quad \frac{D\theta}{Dt} = \frac{\dot{Q}}{C_p}\left(\frac{p_s}{p}\right)^{\kappa}$$

c) Equations of motion

Primitive equations in log pressure coordinate (summary):

Newton law:

$$\frac{Du}{Dt}-\frac{uv}{a}\tan\phi-2\Omega\sin\phi\, v=\frac{-1}{a\cos\phi}\frac{\partial\Phi}{\partial\lambda}+X$$

$$\frac{Dv}{Dt}+\frac{u^2}{a}\tan\phi+2\Omega\sin\phi\, u=\frac{-1}{a}\frac{\partial\Phi}{\partial\phi}+Y$$

$X, Y:$ Body forces like boundary Layer drag

$$0=-\frac{\partial\Phi}{\partial z}+\frac{RT}{H}$$

(Hydrostatic approximation)

Mass conservation:

$$\frac{\rho_0}{a\cos\varphi}\left(\frac{\partial u}{\partial\lambda}+\frac{\partial v\cos\varphi}{\partial\varphi}\right)+\frac{\partial\rho_0 w}{\partial z}=0$$

avec $\rho_0=\rho_s\exp\left(-z/H\right)$

Thermodynamics:

$$\frac{D\theta}{dt}=\frac{\dot{Q}}{C_p}\left(\frac{p_s}{p}\right)^{\kappa}$$ or: $$\frac{DT}{dt}+w\frac{\kappa T}{H}=\frac{\dot{Q}}{C_p}$$

The local buoyancy frequency:

$$N^2=\frac{g}{T_s}\left(\frac{dT}{dz}+\frac{\kappa T}{H}\right)=\frac{T}{T_s}N_*^2$$

Material derivatives:

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+\frac{u}{a\cos\phi}\frac{\partial}{\partial\lambda}+\frac{v}{a}\frac{\partial}{\partial\phi}+w\frac{\partial}{\partial z}$$

Wind components:

$$u=a\cos\phi\frac{D\lambda}{Dt},\quad v=a\frac{D\phi}{Dt},\quad w=\frac{Dz}{Dt}.$$

c) Equations of motion

Boussinesq approximation: $\Phi=\frac{p}{\rho_s}$, $\rho_0=\rho_s(H=\infty)$, but $\frac{R}{H}=\frac{g}{T_s}$ and z=z*

Newton law:

$$\frac{Du}{Dt}-\frac{uv}{a}\tan\varphi-2\Omega\sin\varphi v=\frac{-1}{a\cos\varphi}\frac{\partial\Phi}{\partial\lambda}+X$$

$$\frac{Dv}{Dt}+\frac{u^2}{a}\tan\varphi+2\Omega\sin\varphi u=\frac{-1}{a}\frac{\partial\Phi}{\partial\varphi}+Y$$

X, Y, Z: Body forces like boundary Layer drag

$$\frac{Dw}{Dt}=-\frac{\partial\Phi}{\partial z}+b+Z$$

"Buoyancy:" $b=g\frac{T}{T_s}=g\frac{\theta}{\theta_s}=-g\frac{\rho}{\rho_s}$

The Boussinesq approximation does not need to be hydrostatic

Mass conservation:

$$\frac{1}{a\cos\varphi}\left(\frac{\partial u}{\partial\lambda}+\frac{\partial v\cos\varphi}{\partial\varphi}\right)+\frac{\partial w}{\partial z}=0$$

Thermodynamics:

$$\frac{Db}{dt}=\frac{g\dot{Q}}{C_p T_s}$$

The local buoyancy frequency: $N^2=\frac{g}{T_s}\frac{dT}{dz}$

Material derivatives: $$\frac{D}{Dt}=\frac{\partial}{\partial t}+\frac{u}{a\cos\phi}\frac{\partial}{\partial\lambda}+\frac{v}{a}\frac{\partial}{\partial\phi}+w\frac{\partial}{\partial z}$$

Wind components: $$u=a\cos\phi\frac{D\lambda}{Dt},\quad v=a\frac{D\phi}{Dt},\quad w=\frac{Dz}{Dt}.$$